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DEVELOPING SAFETY PERFORMANCE FUNCTIONS FOR A MOUNTAINOUS FREEWAY

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ABSTRACT

Safety Performance Function (SPF) is essential in traffic safety analysis; it is useful to unveil hazardous factors that related to crash occurrence. Variety data resources have been employed to develop safety performance functions: geometric characteristic features, traffic status information, weather, and surface conditions. This study focuses on a mountainous freeway that features mountainous terrain and adverse weather. Five years crash data (2006-2010), roadway geometry, and traffic characteristics were included in the investigation. As an aggregate analysis, explanatory variables used in the safety performance functions were typically averaged values over a certain time interval. For example, the most applied exposure factor, average annual daily traffic (AADT) is the mean values of the segment annual daily traffic volumes. Moreover, speed limits were included in the models to represent the effects of different traffic characteristics on crash occurrence. However, values of daily volumes and average speeds vary across the whole year, especially for a mountainous freeway suffered from adverse weather and steep slopes. In this study, distributions of daily volume and distributions of 5-min average speed were prepared for each segment. These two distribution variables would be analyzed along with the traditional variables like longitudinal grades, degrees of curvature, and etc to develop SPFs for the mountainous freeway. Data from a 15-mile mountainous freeway on I-70 in Colorado were utilized. Five years crash data have been analyzed along with the Bayesian random effects Poisson model; daily volumes were captured by Remote Traffic Microwave Sensor (RTMS) detectors and segment average speeds were archived by Automatic Vehicle Identification (AVI) systems. Three models have been estimated: (1) ordinal safety performance function with fixed AADT and speed limits; (2) SPF with daily volume distributions; and (3) SPF with both volume and speed distributions. Deviance Information Criterion (DIC), which recognized as Bayesian generalization of AIC (Akaike information criterion) has been select to evaluate the three candidate models. Results indicated that SPFs with distribution variables would improve the model fits significantly.

Keywords: Safety Performance Function; Bayesian inference; Random effects Poisson model; Volume distribution.

1 INTRODUCTION

Motor-vehicle crash studies have been a continuous hot topic in the past decades. Researchers have developed various methods, incorporating different types of data and concluded a variety of countermeasures to improve the highway traffic safety. Efforts have been investigated to identify roadway crash hazardous factors and rank possible improvement treatments. Lord and Mannering (2010) have reviewed and compared different methodologies that were used to develop the safety performance functions. Among the numerous methodologies, random effects Poisson model is one of the most frequent adopted methods. Random effects Poisson models are capable of handling the over-dispersion issue compared to the basic Poisson models; and the modeling results are easy to be interpreted relative to the data mining techniques. And the random effects Poisson model was employed in this study to develop safety performance functions.

Besides the methodologies, different types of data recourses have been employed to develop safety performance functions (SPFs). In addition to the basic geometric characteristic variables and annual average daily traffic (AADT), extended weather related data like visibility, road surface condition index, temperature, and precipitation as well as traffic data such as speed, volume, and occupancy have also been utilized in the analyses. However, as an aggregate analysis, explanatory variables used in the safety performance functions were typically averaged values over a certain time interval. For example, the most typical exposure variable, average annual daily traffic (AADT) is the mean values of the segment traffic volumes. Moreover, speed limits were included in the models to represent the effects of different speed characteristics on crash occurrence. However, factors like daily volume and daily average speed vary across the whole year, especially for a mountainous freeway featured adverse weather and steep slopes. To reflect the variations of these variables, we come up with the thought to utilize additional variables to replace these fixed variables. In the previous study (Yu *et al.*, 2013), we extracted average speeds prior to each crash observation from the real-time traffic data and then aggregate them at segment level; by preparing the dataset in this approach would provide deeper insight of the crash occurrence contributing factors.

In this study, unlike the fixed average values used in the traditional crash frequency models or the previous method that needs to identify every single observation's traffic status, distributions of daily traffic volume and distributions of 5-min average speed were employed to replace the fixed AADT and speed limits variable, respectively. Five years crash data (2006-2010) from a 15-mile mountainous freeway section on I-70 in Colorado were analyzed; for this specific freeway section, Remote Traffic Microwave Sensors (RTMS) detectors and Automatic Vehicle Identification (AVI) systems were implemented along the roadway segments. RTMS detectors provide volume information every 30 seconds while the AVI system archives segment average speed at 1-minute interval. These two types of Intelligent Transportation System detectors enable us to formulate distributions from the real-time traffic data for volume and speed.

In order to handle the mixed explanatory variables with fixed variables along with distribution variables, Bayesian random effects Poisson models were employed in this study. Three models have been estimated: (1) ordinal safety performance function with fixed AADT and speed limits variables; (2) SPF with daily volume distribution variable; and (3) SPF with both daily volume and 5-min average speed distribution variables. Model comparisons have been executed based on the Deviance Information Criterion (DIC), which was recognized as Bayesian generalization of AIC (Akaike information criterion). Finally, benefits of utilizing distribution variables instead of fixed values, and the influences of distribution variables on other explanatory variables would be concluded.

2 BACKGROUNDS

Random effects models have been widely used in crash frequency studies (Shankar *et al.*, 1998; Miaou and Lord, 2003; Guo *et al.*, 2010; Yaacob *et al.*, 2010). Researchers have benefited from their advantages of handling temporal and spatial correlations (Lord and Mannering, 2010).

Shankar et al. (1998) investigated the factors that affect median crossover accidents in Washington State. Random effects negative binomial model (RENB) and cross-sectional negative binomial (NB) model have been compared. The authors concluded that RENB model is only superior to the NB model when spatial and temporal effects are included.

Chin and Quddus (2003) included RENB model to deal with the spatial and temporal effects in the traffic crash study. The authors examined the relationships between accident occurrence and different characteristics of signalized intersections in Singapore. Geometric, traffic, and other control factors were considered in the model. The authors claimed that the random effects have been added to the NB model by assuming that the over-dispersion parameter is randomly distributed across groups, and this formulation is able to account for the unobserved heterogeneity across locations and time.

Traffic variables always play a vital role in crash occurrence studies. Kononov *et al.* (2011) used Annual Average Daily Traffic (AADT) as the only variable to develop the SPF and the results indicated that when some critical traffic densities are reached, the crash occurrence likelihood would increase at a faster rate with an increase in traffic. Besides, in a spatially disaggregate road casualty analysis, Noland and Quddus (2004) used proximate employment variables to represent the different traffic flow scenarios and the results indicated that traffic flow has a high influence on increasing casualties. Furthermore, an AAA foundation for Traffic Safety study (1999) focused on congestion and crashes concluded that a U-shaped model can explain the relationship between the two; crash rates are high at low levels of congestion and rapidly decrease as the volume to capacity (v/c) ratios increase, however they will increase again as the peak levels of congestion turn up. With the help of the data mining method of Classification and Regression Tree (CART), Chang and Chen (2005) concluded that AADT were the key determinant for freeway accident frequencies. However, similar to the weather related variables, using only aggregated traffic data such as AADT would lead to the loss of the most valuable information of pre-crash traffic status.

The Bayesian inference method is a frequently adopted approach to predict crash occurrence in recent studies. A Hierarchical Bayes model was built to estimate area-based traffic crashes (Miaou *et al.*, 2011). Shively *et al.* (2010) employed a Bayesian nonparametric estimation procedure in their study. A 5 X ST-level hierarchy structure was proposed to deal with multilevel traffic safety data (Huang and Abdel-Aty, 2010). Guo *et al.* (2010) included three types of Bayesian models in consideration of different complexities; fixed effect model, mixed effect model and conditional autoregressive (CAR) model have been compared. Furthermore, previous work (Ahmed *et al.*, 2011) on the same freeway segment employed Bayesian hierarchical models to account for seasonal and spatial correlations.

3 DATA PREPARATION

Four datasets were included in this study, (i) I-70 crash data (2006-2010) provided by Colorado Department of Transportation (CDOT), (ii) road segments geometry data obtained from the Roadway Characteristics Inventory (RCI), (iii) real-time traffic data detected by 24 RTMS radars and (iv) real-time traffic data detected by 23 Automatic Vehicle Identification (AVI) detectors located on the east and west bounds along the I-70. The first two datasets were employed to develop the ordinal safety performance function; while the RTMS radars' data were included to develop distributions for the daily volumes and AVI data were used to formulate distributions for 5-min average speeds.

A total of 1123 crashes were documented within the study period. The 15-mile segment, starting at Mile Marker (MM) 205 and ends at MM 220, has been split into 120 homogenous segments (60 in each direction). Given the variations of roadway geometry, homogeneity in roadway alignment was chosen to be the major concern of the segmentation. A minimum length of 0.1 mile was set to avoid the low exposure problem and large statistical uncertainty of the crash rate per short segment (Miaou, 1994). More detailed description of the homogenous segmentation method has been described in a previous study (Ahmed *et al.*, 2011). Table 1 provides descriptive statistics for the significant variables included in the safety performance functions.

Table 1: Summary of variables' descriptive statistics

Variables	**Description**	**Mean**	**Std dev.**	**Min**	**Max**
Dependent Variables					
Crash Frequency	Crash Frequency counts per segment	9.36	10.47	0	58
Independent Variables					
Degree of curvature	Degree of the curve per segment	1.44	1.53	0	4.25
Curve length ratio	Percentage of curve length to total segment length	0.52	0.46	0	1.0
Median Width		25.23	15.26	2	50
Speed Limit		59.3	4.89	50	65
Three lane	1 if three-lane segment;	0.58	0.49	0	1.0

Variables	**Description**	**Mean**	**Std dev.**	**Min**	**Max**
	0 if two-lane segment				
Grade	Longitudinal grade, eight categories: Upgrade: 0-2%=1, 2-4%=2, 4-6%=3, 6-8%=4; Downgrade: 0-(-2)%=5, (-2)-(-4)%=6, (-4)-(-6)%=7, (-6)-(-8)%=8	4.45	2.40	1	8
Exposure Variables					
LogAADT	Logarithmic transformation of segment AADT	10.26	0.06	10.14	10.28
LogLength	Logarithmic transformation of segment length	-1.59	0.54	-2.38	-0.08

For the RTMS detectors, information about speed, volume, and occupancy was archived at 30-second interval. In order to achieve daily volume distributions, raw RTMS data have been aggregated into daily volumes and then these data were analyzed by PROC SEVERITY procedure in SAS (SAS Institute, 2004). The procedure would select the best fitted distribution for the daily volume from a set of candidate distributions. Normal, Gamma, Exponential, Lognormal, and Weibull distribution are the candidate distributions in this study and the Maximum likelihood method was used to estimate the parameters of the distributions in the procedure. Moreover, likelihood-based statistics were supplied to indicate the data fits of the estimated distributions. Among the likelihood based statistics Akaike's information criterion (AIC) was selected to identify the most appropriate distributions for the variables. The smaller the AIC value is, the better the distribution fits the data. The results indicate that Gamma distribution fits the daily volume distribution best (see Figure 1 and Table 2 for example).

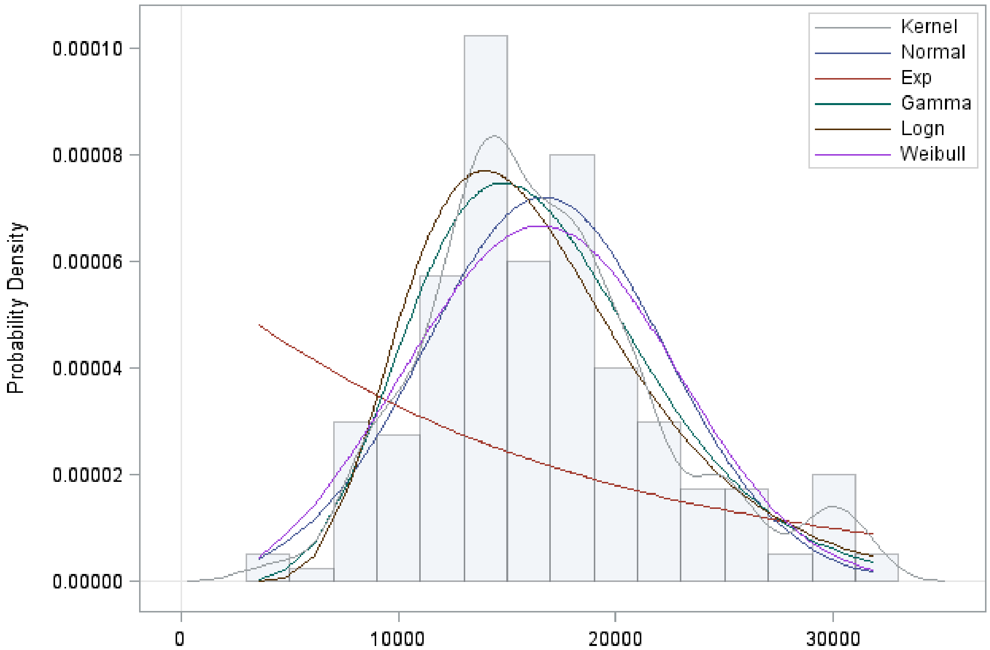

Figure 1: Distribution fits for daily volume

Table 2: Distribution fitting and selection results

Distribution	Converged	AIC	Selected
Normal	Yes	4019	No
Exponential	Yes	4291	No
Gamma	Yes	4008	Yes
Lognormal	Yes	4017	No
Weibull	Yes	4018	No

Besides, in order to formulate distributions for the 5-minute average speeds, AVI data were chosen instead of the RTMS data. The reason that AVI speed was preferred is that AVI data provide segment average speeds which reflect the traffic status along a certain roadway segment; while the RTMS speeds are point detected speeds which are highly correlated to the detector locations and would be bias if used to represent segment traffic status. Moreover, AVI data have been proved to be useful in measuring real-time traffic safety in our previous studies (Ahmed and Abdel-Aty, 2011; Ahmed *et al.*, 2012). The raw AVI data (1-minute interval) have been aggregated into 5-minute and the 5-minute average speeds have been calculated. Same procedures as developing distributions for daily volume have been conducted; it was concluded that Weibull distribution best represents the speed distributions (see Figure 2 for example).

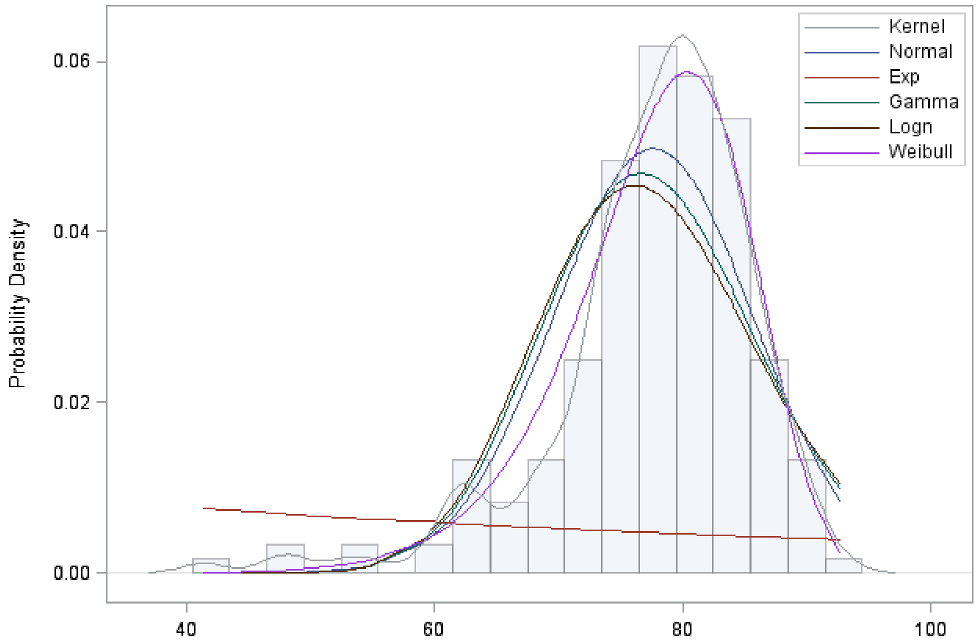

Figure 2: Distribution fits for 5-min average speed

4 METHODOLOGY

Crash occurrence along the freeway can be assumed to follow Poisson process. The Poisson model has played a key role in crash-frequency studies. However it has been blamed of lacking the ability to handle over-dispersion problems (Lord and Mannering, 2010). Multiplicative gamma distributed random effects were introduced into the Poisson model, which implies a negative binomial marginal sampling distribution (Ntzoufras, 2009). The random effects Poisson model can be setup as follows:

$$Y_i \sim Poission(\lambda_i)$$
$$log\lambda_i = \ loge_i + \boldsymbol{X_i \beta} + \ u_i + \ \gamma_1 a[i] + \gamma_2 b[i]$$

where Y_i is the crash count at segment i (i=1, . . . , 120 (60 segments on each direction)) during. $\boldsymbol{X_i}$ represent the risk factors, $\boldsymbol{\beta}$ is a vector of regression parameters and u_i is the segment specified random effects. a and $b$are detector indexes for the RTMS and AVI detectors while γ_1 and γ_2 are the coefficients for the volume and speed distribution variables, respectively. For each detector, specific distributions have been assigned with the results from the data preparations:

$$a_t \sim gamma\ (a, b), for\ t = 1, \ldots, 24$$
$$Mean = \frac{a}{b}, Variance = \frac{a}{b^2}$$
$$b_t \sim weibull\ (v, \lambda), for\ t = 1, \ldots, 23$$

$$Mean = \lambda^{-\frac{1}{v}}\Gamma(1+v^{-1}), Variance = [\Gamma(1+2v^{-1}) - \Gamma(1+v^{-1})^2]\lambda^{-2/v}$$

Full Bayesian inference was employed in this study, for each model, three chains of 15,000 iterations were set up in WinBUGS (Lunn *et al.*, 2000), 5,000 iterations were used in the burn-in step.

5 MODELING RESULTS AND DISCUSSIONS

Table 3 provides the parameter estimations, credible intervals, and model fit for the random effects Poisson model with fixed variables. Degree of curvature variable is significant with a negative sign, which indicates that segments with a sharp curve are less likely to have crashes relative to the flat curves. It is not surprising that higher degree of curvature was concluded as associated with lower crash likelihood (Shankar *et al.*, 1995; Anastasopoulos *et al.*, 2008), drivers seem driving more cautious during sharp curves. Curve length ratio variable represents the percentages of curve length to total segment length, which has a positive sign; this demonstrates that segments with longer curves are prone to have more crashes. The three lane indicator is proved to be negatively associated with high crash frequency which indicates that fewer crashes occurred at three lane segments. The median width variable is also significant with a negative sign, which demonstrates that larger medians could reduce crash occurrence. In addition, the longitudinal grade variables are significant (reference to the Grade [8], downgrade slopes range from 6% to 8%): generally the steeper the slope, the higher crash risk; segments with downgrade slopes are relatively more hazardous than the corresponding upgrades with same slope ranges. Furthermore, the segment length is the only significant exposure parameter; this is because that for the 120 segments, only few AADT values were extracted from the RCI data which cannot represent the variations of daily volume.

Table 3: Parameter estimates for the random effects Poisson model

Variables	**Mean**	**Std. dev**	**2.5%**	**97.5%**
Degree of curvature	-0.17	0.038	-0.24	-0.09
Curve length ratio	0.55	0.11	0.35	0.76
Median width	-0.016	0.003	-0.02	-0.01
Three lane	-0.33	0.08	-0.49	-0.16
Speed limit	0.0038	0.0069	-0.009	0.017
LogLength	0.99	0.06	0.87	1.11
LogAADT	-0.055	0.14	-0.34	0.21
Grade G[1]	-1.49	0.21	-1.91	-1.09
Grade G[2]	-0.22	0.12	-0.46	0.008
Grade G[3]	-0.51	0.09	-0.69	-0.34
Grade G[4]	-0.32	0.12	-0.56	-0.074
Grade G[5]	-1.29	0.17	-1.63	-0.97
Grade G[6]	-0.26	0.15	-0.55	0.03
Grade G[7]	-0.45	0.12	-0.7	-0.21

Variables	Mean	Std. dev	2.5%	97.5%
Grade G[8] (Reference)	-	-	-	-
DIC	770.648			

In addition to the basic random effects Poisson model with traditional explanatory variables, two extra SPFs with volume distribution variable and both volume and speed distribution variables have been estimated. Table 4 shows the parameter estimations and goodness-of-fit for the two extra models. After replacing the fixed AADTs with volume distributions, Log Volume distribution variable is significant with a positive sign which demonstrates that larger volumes would lead to more crash occurrence due to the bigger exposure. Furthermore, segment speed distributions have been included to replace the fixed speed limits to reflect various traffic conditions. Same as the speed limit variable, a positive sign has been concluded for the speed distribution variable, which indicates higher speeds are positively related to higher crash rates. However, after introducing the speed distributions, some of the longitudinal grade variables and three lane indicator variable became insignificant which can be understood as that speed distributions varies across different geometry features; those steep slopes effects on crash occurrence might be represented by the speed distributions.

For the model comparisons, the DIC, recognized as Bayesian generalization of AIC, was used to compare the performances of the three candidate models. DIC is a widely used evaluation measure for Bayesian models, according to (Spiegelhalter *et al.*, 2003), differences of more than 10 might definitely rule out the model with higher DIC. Differences between 5 and 10 are considered substantial. It can be detected that after incorporating the distribution variables; model fit has been substantially improved and the model with both speed and volume distribution has the best goodness-of-fit.

Table 4: Parameter estimates for the models with distribution variables

	SPF with volume distribution				**SPF with both speed and volume distributions**			
Variables	Mean	Std. dev	2.5%	97.5%	Mean	Std. dev	2.5%	97.5%
Degree of curvature	-0.16	0.038	-0.24	-0.08	-0.17	0.04	-0.25	-0.09
Curve length ratio	0.51	0.11	0.29	0.71	0.56	0.11	0.34	0.78
Median width	-0.008	0.003	-0.015	-0.002	-0.01	0.004	-0.018	-0.001
Three lane	-0.29	0.083	-0.46	-0.13	-0.052	0.14	-0.31	0.24
Speed limit	0.017	0.008	0.0006	0.032	-	-	-	-
Speed distribution	-	-	-	-	0.028	0.008	0.011	0.049
LogLength	0.94	0.059	0.82	1.06	0.92	0.066	0.79	1.05
Log Volume distribution	0.57	0.17	0.28	0.91	0.44	0.19	0.18	0.91
Grade G[1]	-1.51	0.20	-1.92	-1.12	-0.88	0.33	-1.51	0.19

	SPF with volume distribution				SPF with both speed and volume distributions			
Grade G[2]	-0.39	0.12	-0.65	-0.14	-0.11	0.19	-0.47	0.29
Grade G[3]	-0.58	0.09	-0.77	-0.41	-0.15	0.21	-0.52	0.30
Grade G[4]	-0.24	0.13	-0.49	0.008	0.31	0.25	-0.14	0.84
Grade G[5]	-1.33	0.17	-1.66	-1.01	-0.89	0.28	-1.43	-0.34
Grade G[6]	-0.29	0.15	-0.59	-0.009	-0.13	0.20	-0.52	0.26
Grade G[7]	-0.41	0.12	-0.65	-0.17	-0.33	0.14	-0.60	-0.54
Grade G[8] (Reference)	-	-	-	-	-	-	-	-
DIC	760.251				730.797			

6 CONCLUSIONS

Safety performance functions are widely employed to identify the hazardous factors and also the black spots recognized as high crash rates locations. This study focused on a mountainous freeway section of I-70 in Colorado; five years crash data have been analyzed along with the geometry characteristics and traffic conditions. Firstly, an ordinal SPF has been estimated with Bayesian random effects Poisson model and the traditional explanatory variables. The modeling results demonstrate that degrees of curvature, median widths are negatively related to crash occurrence while curve length ratio and speed limits were proved to be positively associated with more crashes. In addition, more crashes were found occurred at two lane segments and the larger the homogenous segment, the more crashes would be expected.

However, due to limitations of the data, only few values of AADT were assigned within the 120 homogenous segments; which turned out that AADT is not significant in the developed safety performance function. For the sake of reflecting the actual variations of daily volumes and average speeds, two distribution variables have been included in the SPFs. Distributions of daily volume and average speed were benefited from the implemented ITS detectors along the roadway section; whereas RTMS detectors' data were used to formulate distributions for daily volumes while distributions of average speed were developed from AVI segment speed data. The introduction of the distribution variables was proved to be capable of improving model goodness-of-fits and certain variable's significance (AADT). Nonetheless, after employing speed distributions instead of the speed limits, longitudinal grade variables became insignificant which needs further investigations.

Moreover, the results presented in this paper are based on the particular data from a mountainous freeway, which is somewhat unique. Further researches with different data resources and infrastructure types are needed to confirm the results concluded in this study.

REFERENCES

AAA Foundation for Traffic Safety. 1999. Did You Know? Congestion and Crashes. http://www.aaafoundation.org/pdf/congest.pdf. Accessed 2012-03-25.

Ahmed, M., Abdel-Aty, M., Yu, R., 2012. Assessment of the interaction between crash occurrence, mountainous freeway geometry, real-time weather and avi traffic data *Journal of Transportation Research Board*, In Press.

Ahmed, M., Huang, H., Abdel-Aty, M., Guevara, B., 2011. Exploring a bayesian hierarchical approach for developing safety performance functions for a mountainous freeway. *Accident Analysis and Prevention*, Vol.43, pp.1581-1589.

Ahmed, M., Abdel-Aty, M., 2011. The viability of using automatic vehicle identification data for real-time crash prediction. *Intelligent Transportation Systems, IEEE Transactions*, Vol.99, pp.1-10.

Anastasopoulos, P., Tarko, A., Mannering, F., 2008. Tobit analysis of vehicle accident rates on interstate highways. *Accident Analysis and Prevention*, Vol.40, pp.768-775.

Chang, L., Chen, W., 2005. Data mining of tree-based models to analyze freeway acident frequency. *Journal of Safety Research*, Vol.36, pp.365-375.

Chin, H., Quddus, M., 2003. Applying the random effect negative binomial model to examine traffic accident occurrence at signalized interesections. *Accident Analysis and Prevention*, Vol.35, pp.253-259.

Guo, F., Wang, X., Abdel-Aty, M., 2010. Modeling signalized intersection safety with corridor-level spatial correlations. *Accident Analysis and Prevention*, Vol.42, pp.84-92.

Huang, H., Abdel-Aty, M., 2010. Multilevel data and bayesian analysis in traffic safety. *Accident Analysis and Prevention*, Vol.42, pp.1556-1565.

SAS Institute, 2004. Sas/graph (r) 9.1 reference Sas Inst.

Kononov, J., Lyon, C., K.Allery, B., 2011. Relating flow, speed and density of urban freeways to functional form of an SPF. *Compendium of papers CD-ROM,TRB Annual Meeting 2011. Washington, D.C*.

Lord, D., Mannering, F., 2010. The statistical analysis of crash-frequency data: A review and assessment of methodological alternatives. *Transportation Research Part A: Policy and Practice*, Vol.44, pp.291-305.

Lunn, D., Thomas, A., Best, N., Spiegelhalter, D., 2000. Winbugs-a bayesian modelling framework: Concepts, structure, and extensibility. *Statistics and Computing*, Vol.10, pp.325-337.

Miaou, S., Lord, D., 2003. Modeling traffic crash-flow relationships for intersections: Dispersion parameter, functional form, and bayes versus empirical bayes. *Transportation Research Record*, Vol.1840, pp.31-40.

Miaou, S.., Song, J., Mallick, B., 2011. Roadway traffic crash mapping: A space-time modeling approach. *Journal of Transportation and Statistics*, Vol.6, No.3.

Miaou, S., 1994. The relationship between truck accidents and geometric design of road sections: Poisson versus negative binomial regressions. *Accident Analysis and Prevention*, Vol.26, pp.471-482.

Noland, R., Quddus, M., 2004. A spatially disaggregate analysis of road casualties in england. *Accident Analysis and Prevention*, Vol.36, pp.973-984.

Ntzoufras, I., 2009. Bayesian modeling using winbugs WILEY.

Shankar, V., Albin, R., Milton, J., Mannering, F., 1998. Evaluating median crossover likelihoods with clustered accident counts: An empirical inquiry using the random effects negative binomial model. *Transportation Research Record*, Vol.1635, pp.44-48.

Shankar, V., Mannering, F., Barfield, W., 1995. Effect of roadway geometrics and environmental factors on rural freeway accident frequencies. *Accident Analysis and Prevention*, Vol.27, pp.371-389.

Shively, T., Kockelman, K., Damien, P., 2010. A bayesian semi-parametric model to estimate relationships between crash counts and roadway characteristics. *Transportation Research Part B*, Vol.44, pp.699-715.

Spiegelhalter, D., Thomas, A., Best, N., Lunn, D., 2003. Winbugs user manual. Cambridge: MRC Biostatistics Unit.

Yaacob, W., Lazim, M., Wah, Y., 2010. Evaluating spatial and temporal effects of accidents likelihood using random effects panel count model. *In: Proceedings of the 2010 International Conference on Science and Social Research, Kuala Lumpur, Malaysia*.

Yu, R., Abdel-Aty, M., Ahmed, M., 2013. Bayesian random effect models incorporating real-time weather and traffic data to investigate mountainous freeway hazardous factors. *Accident Analysis and Prevention* Vol.50, pp.371-376.