

**SIXTH FRAMEWORK PROGRAMME
PRIORITY 1.6.2
Sustainable Surface Transport**



CATRIN

Cost Allocation of TRansport INfrastructure cost

D8 – Rail Cost Allocation for Europe

**Annex 1A – Marginal Cost of Railway Infrastructure
Wear and Tear for Freight and Passenger Trains in
Sweden**

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CATRIN Partner Organisations

VTI; University of Gdansk, ITS Leeds, DIW, Ecoplan, Manchester Metropolitan University, TUV
Vienna University of Technology, EIT University of Las Palmas; Swedish Maritime Administration,
University of Turku/Centre for Maritime Studies

CATRIN D8 - Rail Cost Allocation for Europe – Annex 1A - Marginal Cost of Railway Infrastructure Wear and Tear for Freight and Passenger Trains in Sweden

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Project summary

CATRIN is a Research project to support the European Transport Policy, specifically to assist in the Implementation of Transport pricing. CATRIN will increase the probability that new progressive pricing principles can be implemented which facilitate a move towards sustainable transport. CATRIN is both intermodal and interdisciplinary, emphasize the need of new Member states, understands that different organisational forms require different recommendations, that recommendations need to be given in short and long-term perspective and that they have to be thoroughly discussed with infrastructure managers.

CATRIN will clarify the current position on allocation of infrastructure cost in all modes of transport. Pricing principles will be dealt with under the knowledge that they varies with the organisational structure of a sector. CATRIN will establish the micro-aspects of cost recover above marginal costs, including the results of applying a club approach and the implication of who bears the costs for cost recovery under alternative allocation rules, using game theoretic analytical tools.

CATRIN will develop the understanding of policy need of new Member states and can give tailored recommendations. In a modal focus, with real world cases, CATRIN will develop proxies to marginal costs and test some of the allocation approaches. Based on engineering studies CATRIN will analyse the possibility to defining more differentiated pricing rules for vehicle/locomotive categories. Partners with strong engineering knowledge are included and CATRIN will blend the economic principles of pricing with engineering knowledge. CATRIN will outline the possibilities for a European Road Damage test that will give new evidences on the fourth-power-rule. CATRIN will develop financing alternatives for icebreaking and will explore cost allocation in the aviation sector. Finally, CATRIN will strongly address the implementation potential and constraints experienced by infrastructure managers.

Marginal cost of railway infrastructure wear and tear for freight and passenger trains in Sweden

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Abstract

We analyse maintenance cost data for Swedish railway infrastructure in relation to traffic volumes and other characteristics, and separate the cost impact from passenger and freight trains. Lines with mixed passenger and freight traffic, and dedicated freight lines are analysed separately using both log-linear and Box-Cox regression models. We find that for mixed lines, the Box-Cox specification is preferred, while a log-linear model is chosen in the case of dedicated freight lines. The cost elasticity with respect to output is found to be higher for passenger trains than for freight trains. From a marginal cost pricing perspective, freight trains are currently paying too much, while passenger trains should be charged more. An adjusted pricing scheme based on these results would still lead to higher revenues than today if total demand is unaffected.

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1 Introduction

There has been increasing European attention to the issue of marginal costs of railway infrastructure wear and tear in the last decade. European rail infrastructure administrations have great interest in these marginal cost estimates as they are an important corner-stone of the European transport pricing policy (European Parliament, 2001). The EU-project CATRIN (Cost Allocation of TRansport INfrastructure cost) supports the European Transport Policy, specifically to assist in the implementation of transport pricing for all modes of transport. Following the paper by Johansson and Nilsson (2004) on railway infrastructure maintenance costs, there is now research ongoing in several European countries (Lindberg, 2006).

The general approach is to do regression analysis on maintenance costs and control for infrastructure characteristics and traffic volumes. The majority of recent studies use an aggregate measure of output of the track, which is expressed in total gross tonnes of traffic consisting of both passenger and freight trains. Furthermore, log-linear models are dominating the research.

The Swedish Rail Administration (Banverket) is responsible for railway access charges in Sweden. The current charge for infrastructure wear and tear is SEK 0.0029 per gross tonne kilometre as a flat rate for all users (Banverket, 2008).¹ To increase efficiency in current pricing schemes, introducing differentiated track access charges has been discussed, based on wear and tear from different vehicle types. The hypothesis is that freight and passenger trains deteriorate the infrastructure differently, inducing different levels of cost and therefore should be priced accordingly. The reason for this position is that freight and passenger trains generate different forces on the railway track through differences in speeds, axle loads, suspensions etcetera as well as require different track quality levels. This issue has also received some attention in Sweden in a report on differentiated access charges by track engineers at the Royal Institute of Technology (KTH) and Banverket (Öberg et al., 2007).

Whether this standpoint can be supported by empirical, econometric work is yet to be revealed, but some preliminary work by Gaudry and Quinet (2003) indicates that there might be substantial differences in wear and tear, not only between freight and passenger trains, but

¹ The exchange rate between Swedish Kronor (SEK) and Euro is 10.18 (PPP estimate from year 2001).

also within the group of passenger trains. Furthermore, they advocate in favour of the Box-Cox model as an alternative to previously used log-linear models. To be able to analyse the question of differentiation, the aggregate measure of traffic volume has to be abandoned in favour of a model where the different traffic categories are used as outputs.

In this paper, we analyse a four-year data set on Swedish railway maintenance costs in order to contribute to the analysis on differentiated marginal costs. The purpose is threefold. First, we are interested in separating gross tonnes for freight and passenger trains in order to see if cost elasticities and marginal costs are different for the two traffic categories. Second, the choice between logarithmic and Box-Cox transformation of the data will be analysed. Third, lines with a mixed passenger and freight traffic pattern will be separated from lines dedicated to freight traffic only to see if there are systematic differences in freight marginal costs between these track types.

The paper is structured as follows. A short overview of recent work is given in section 2 followed by a description of the data in section 3. Model specifications and results from the econometric analyses with marginal cost calculations are given in section 4 and 5 respectively. In section 6, we discuss our results and draw conclusions.

2 Literature review

The issue of estimating cost functions for railway organisations has a long history and can be found as early as the 1960's (Borts, 1960). The focus of the early research was to check for inefficiencies in the U.S. railroad industry and to regulate monopoly prices in the presence of economies of scale (Keeler, 1974).

Recent European studies have a different perspective as they are looking at the cost structure in vertically separated rail infrastructure organisations to derive short run marginal costs. These studies have grown out of a sequel of research projects on transport infrastructure pricing funded by the European Commission, such as Pricing European Transport Systems (PETS) (Nash and Sansom, 2001), UNification of accounts and marginal costs for Transport Efficiency (UNITE) (Nash, 2003) and Generalisation of Research on Accounts and Cost Estimation (GRACE) (Nash et al., 2008).

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The study that initiated most of the current work is Johansson and Nilsson (2004) who estimate rail infrastructure maintenance cost functions on data from Sweden and Finland from the mid 1990's. They apply a reduced form of the Translog specification suggested by Christensen et al. (1973) using total gross tonnes as output of the track, controlling for infrastructure characteristics, but excluding factor prices. The analysis builds on the assumption that costs are minimised for a given level of output. Cost elasticities and marginal costs are given as main results.

Railway infrastructure maintenance cost functions have since then been estimated in Austria (Munduch et al., 2002), Norway (Daljord, 2003), Finland (Tervonen and Idström, 2004), Switzerland (Marti and Neuenschwander, 2006), Sweden (Andersson, 2006, 2007a and 2008) and the UK (Wheat and Smith, 2008). All of these studies use log-linear model specifications and also an aggregate measure of output, i.e. total gross tonnes. Pooling annual data for several years is done in all cases, except for Andersson (2007a and 2008) who uses panel data techniques.

Considering the variation between the individual studies, the results have been reasonably similar in terms of cost elasticities with respect to output, when controlling for the cost base included (Wheat, 2007). There is evidence for the maintenance cost elasticity with respect to output of gross tonnes to be in the range of 0.2 - 0.3, i.e. a 10 percent change in output gives rise to a 2 - 3 percent change in maintenance costs. Marginal costs on the other hand vary between countries and are more difficult to compare.

The only alternative econometric approaches so far to the one suggested by Johansson and Nilsson (2004) are found in Gaudry and Quinet (2003) and Andersson (2007b). Gaudry and Quinet (2003) use a very large data set for French railways in 1999, and explore a variety of unrestricted generalised Box-Cox models to allocate maintenance costs to different traffic classes. They reject the Translog specification as being too restrictive on their data set, which indicates that a logarithmic transformation of the data is not as efficient as using a Box-Cox transformation. Andersson (2007b) uses survival analysis on rail renewal data to derive marginal costs.

3 The data

The available data set consists of some 185 track sections with traffic (freight and/or passenger) that we observe over the years 1999 - 2002. A track section is a part of the network, normally a link between two nodes or stations that varies in length and design. Each track section observation has information on annual maintenance costs (ccm_{tot})², traffic volumes expressed as gross tonnes for freight (fgt) and passenger trains (pgt) as well as a range of infrastructure characteristics. These are track length (bis_{tsl}), track section length-to-distance ratio (ld_{ratio}), length of switches ($swit_{tl}$), average rail age ($rail_{age}$), average switch age ($swit_{age}$), number of joints ($joints$), average rail weight ($rlwgh$) and average quality class (qc_{ave}). Maintenance costs are derived from Banverket's financial system and cover all maintenance activities. Both corrective and preventive maintenance is included, but winter maintenance (snow clearing and de-icing) is excluded. Major renewals are also excluded, but it might include minor replacements considered as spot-maintenance. A description of the cost data is given in Appendix 1. Infrastructure characteristics are taken from the track information system at Banverket and traffic volumes from various Swedish train operating companies.

We have split the original data set into two parts. One part is tracks with mixed traffic and the other is tracks dedicated to freight trains only. The reason for this is the underlying idea behind the marginal cost calculation and differentiation. Tracks without any passenger traffic are significantly different from tracks with mixed traffic from an engineering point of view. This has to do with the alignment and design of the track to deal with different train types running at different speeds with different loads. A dedicated freight line can be aligned to minimise deterioration and cost from a freight train, while the alignment for a mixed line has to be a compromise between the needs for both freight and passenger trains. In a mixed situation, freight trains will normally run at lower speeds and weights than passenger trains leading to freight trains “hanging” on the inner rail in curves, while passenger trains will “push” towards the outer rail. The super-elevation (cant) of the track is therefore non-optimal for both. Introducing a marginal change in passenger traffic (running the first passenger train) on a dedicated freight line would therefore not give rise to a marginal change in costs, but rather a leap in costs to adjust the alignment to the mixed situation as well as covering the

² Costs are expressed in Swedish kronor (SEK) and 2002 price level.

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costs from the passenger train. Our position is that dedicated lines are better off to be analysed separately and these results will be presented alongside results of mixed lines.

The mixed line data set covers 648 observations, i.e. around 160 track sections over four years, and our dedicated freight line data set contains 101 observations (around 25 track sections).

Table 1. Descriptive statistics

Variable	No. Obs.	MEAN	ST. DEV.	MIN.	MAX.	+/-
<u>Mixed lines</u>						
<i>ccm_tot</i>	648	7,650,672.00	7,775,205.00	130,530.00	80,852,300.00	n.a.
<i>fgt</i>	648	5,349,595.00	8,007,622.00	6,426.95	85,571,500.00	+
<i>pgt</i>	648	3,096,828.00	5,116,585.00	74.72	46,913,700.00	+
<i>bis_tsl</i>	648	74,589.15	55,515.31	3,719.00	261,561.00	+
<i>ld_ratio</i>	648	1.92	1.50	1.00	11.01	-
<i>swit_tl</i>	648	1,855.96	1,785.92	58.03	14,404.70	+
<i>rail_age</i>	648	17.21	9.59	2.00	60.66	+
<i>swit_age</i>	648	17.63	8.64	1.00	45.25	+
<i>joints</i>	648	168.74	134.29	1.00	799.00	+
<i>rlwgh</i>	648	50.87	4.60	39.77	60.00	-
<i>qc_ave</i>	648	2.06	1.05	0.00	4.59	+
<i>cost/track metre</i>	648	115.49	84.05	5.89	667.47	n.a.
<i>cost/gross tonne</i>	648	2.92	5.99	0.01	73.27	n.a.
<i>cost/gross tonne km</i>	648	0.07	0.09	0.001	0.63	n.a.
<u>Dedicated freight lines</u>						
<i>ccm_tot</i>	101	3,027,278.00	3,636,412.00	54,394.60	24,491,800.00	n.a.
<i>fgt</i>	101	1,027,368.00	1,841,278.00	6,426.95	9,500,550.00	+
<i>pgt</i>	101	n.a.	n.a.	n.a.	n.a.	n.a.
<i>bis_tsl</i>	101	48,984.92	40,238.06	8,878.00	170,162.00	+
<i>ld_ratio</i>	101	1.16	0.34	1.01	2.81	-
<i>swit_tl</i>	101	609.09	411.23	66.46	1,694.19	+
<i>rail_age</i>	101	28.05	23.38	1.00	98.00	+
<i>swit_age</i>	101	26.41	12.22	5.00	67.66	+
<i>joints</i>	101	69.61	60.57	0.00	266.00	+
<i>rlwgh</i>	101	44.79	4.90	32.00	60.00	-
<i>qc_ave</i>	101	3.54	0.64	1.44	4.94	+
<i>cost/track metre</i>	101	63.70	76.92	1.23	656.72	n.a.
<i>cost/gross tonne</i>	101	7.89	11.03	0.18	88.26	n.a.
<i>cost/gross tonne km</i>	101	0.30	0.59	0.004	5.10	n.a.

A descriptive summary of the data sets is given in table 1 and there are some differences between the two data sets worth pointing out:

- Average annual spending on maintenance is close to 2.5 times higher on mixed lines.
- Average freight traffic density is 5 times higher on mixed lines.
- Track length is 1.5 times higher on mixed lines.
- There are 3 times more switches on mixed lines.
- Both switches and rails on dedicated freight lines are on average more than 10 years older than on mixed lines.

- Average track quality is much lower on dedicated freight lines.
- Maintenance costs per gross tonne are almost 3 times higher on dedicated freight lines.
- Maintenance costs per track metre are 2 times higher on mixed lines.

The +/- column indicates our a priori expectation about the relationship between each variable and maintenance costs. Hence, higher values of freight and passenger gross tonnes, track section length, switches, rail and switch age, joints and quality class³ are expected to increase maintenance costs, other things equal. A higher length-to-distance ratio means easier access to the track and would lead to more efficient work schedules and reduced costs. Higher rail weight resists wear and tear and lead to less maintenance.

4 The econometric approach

We have pointed out above that knowledge of marginal costs is essential to European railway administrations. Among the available methods to estimate the marginal costs, we will use an econometric approach, i.e. an application of statistical methods to economic data. To estimate a cost function, we build on the duality between production and costs under the assumption that costs are minimised for a given level of output and input of factor prices.

We can describe the relationship between maintenance costs (C), a vector of outputs (\mathbf{q}) and a vector of factor prices (\mathbf{p}) as

$$C = f(\mathbf{q}, \mathbf{p})$$

For our analyses, we have reasons to believe that the spatial variation in factor prices, i.e. labour, energy and capital costs over the Swedish rail network is negligible. This idea was first suggested by Johansson and Nilsson (2004) with the argument that the Swedish labour market agreements are heavily regulated at a national level. Another reason is that the majority of the track work during these years is done in-house by the Production Division of Banverket. We will therefore exclude the factor price vector \mathbf{p} in our estimated cost functions and proceed with the assumption of equal factor prices over the network.

³ Quality class ranges from 0 (high quality) to 5 (low quality) and can vary over a track section. This is important to note for interpreting results later in the paper.

However, output in terms of traffic volumes is not the only factor that can influence the variation in costs over a rail network. As output varies over the network, so do the technical characteristics of the track, climate and managerial skills, which need to be controlled for. Thus, we will assume that there is a relationship between costs for infrastructure maintenance (C), and the level of output (\mathbf{q}) given other characteristics of the infrastructure (\mathbf{x}) and dummy variables (\mathbf{z});

$$C = f(\mathbf{q}, \mathbf{x}, \mathbf{z}).$$

A **log-linear regression model** in form of this relationship is given in expression (1), where i denote observations, t time, k , m and n are the number of output, infrastructure and dummy variables respectively in the model. α , β_k , δ_m and γ_n are parameters to be estimated. ε is the error term assumed NID $(0, \sigma)$.

$$\ln C_{it} = \ln \alpha + \ln q_{kit} \beta_k + \ln x_{mit} \delta_m + z_{nit} \gamma_n + \varepsilon_{it} \quad (1)$$

The cost elasticity in the log-linear model is the derivative of the cost function with respect to the variable of interest. If the model does not include higher-order or interaction terms, the k elasticities for our output variables are expressed in general form as

$$\frac{\partial \ln C}{\partial \ln q_k} = \hat{\beta}_k = \hat{\phi}_k^{LL}. \quad (2)$$

These elasticities are constant over the range of output we analyse, but including higher order terms or interactions will lead to non-constant elasticities. Exact elasticity expressions will be given under the detailed specifications in the following chapter.

The log-linear model above imposes a restriction on our model as it assumes that the most efficient transformation of our data is logarithmic. An alternative to the logarithmic transformation is the **Box-Cox regression model**, making use of the formula for variable transformation by Box and Cox (Greene, 2003).

$$w^{(\lambda)} = \frac{w^\lambda - 1}{\lambda} \quad (3)$$

For λ to be defined for all values, w must be strictly positive. The direct benefit of using the Box-Cox transformation is that it includes the log transformation as a special case. Hence, if our data are log normal, the transformation parameter λ will be insignificant from zero. If not, the log transformation in model (1) will not be an efficient way of treating our data.

The econometric specification in general form, using a common transformation parameter for both the left and right hand side is given in (4)

$$C_{it}^{(\lambda)} = \alpha + q_{kit}^{(\lambda)} \beta_k + x_{mit}^{(\lambda)} \delta_m + z_{nit} \gamma_n + \varepsilon_{it} \cdot \quad (4)$$

Output (q) and infrastructure (x) variables are transformed, while the constant, variables with genuine zeros and dummy variables (z) are left un-transformed. The elasticity in the Box-Cox model (4) also includes the estimated transformation parameter λ (Econometric Software, Inc., 2002) and the general expression is

$$\frac{\partial \ln C_{it}}{\partial \ln q_{kit}} = \hat{\beta}_k \left(\frac{q_{kit}}{C_{it}} \right)^\lambda = \hat{\phi}_{kit}^{B-C} \cdot \quad (5)$$

Hence, the elasticity in a Box-Cox model will be non-constant and vary with output and cost level. For a derivation of the elasticity, see Appendix 2.

5 Econometric specifications and results

In this section, we present the econometric specifications and results, including elasticities and marginal costs calculations. We start by looking at a model for mixed lines followed by a dedicated freight line model. All estimations are done in *Stata 9* (StataCorp, 2005).

5.1 Mixed lines

As the Box-Cox model includes the log-linear model as a special case, we have initially estimated a Box-Cox regression model on all track sections with mixed traffic (648 observations). The model includes output of both freight (fgt) and passenger (pgt) gross

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tonnes per annum. Apart from that, we control for length-distance ratio (*ld_ratio*), track section length (*bis_tsl*), switches (*swit_tl*), rail age (*rail_age*) and switch age (*swit_age*). These are all transformed variables. Non-transformed variables are joints (*joints*), average quality class (*qc_ave*) and dummy variables for 3 years, 15 track districts and stations. The model specification is given below (6) and the estimated model in table 2 (dummy variables excluded).

$$C_{it}^{(\lambda)} = \alpha + \beta_1 fgt_{it}^{(\lambda)} + \beta_2 pgt_{it}^{(\lambda)} + \beta_3 ld_ratio_{it}^{(\lambda)} + \beta_4 bis_tsl_{it}^{(\lambda)} + \beta_5 swit_tl_{it}^{(\lambda)} + \beta_6 rail_age_{it}^{(\lambda)} + \beta_7 swit_age_{it}^{(\lambda)} + \beta_8 qc_ave_{it} + \beta_9 joints_{it} + \sum_{n=1}^3 \gamma_n year_{it} + \sum_{m=1}^{15} \eta_m district_{it} + \omega_1 station_{it} + \varepsilon_{it} \quad (6)$$

Table 2. Box-Cox regression model estimates – Mixed lines

Box-Cox Regression		Number of obs	=	648	
Log likelihood = -10326.475		LR chi2(28)	=	1095.67	
		Prob > chi2	=	0.000	

ccm_tot	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
/lambda	.1694008	.0209993	8.07	0.000	.1282429 .2105587

Estimates of scale-variant parameters					
	Coef.	chi2(df)	P>chi2(df)	df of chi2	

Notrans					
qc_ave	1.237109	10.875	0.001	1	
joints	.0081398	7.742	0.005	1	
_cons	-10.45406				

Trans					
fgt	.059676	14.466	0.000	1	
pgt	.2235988	94.018	0.000	1	
ld_ratio	-4.468812	32.894	0.000	1	
bis_tsl	1.400582	178.966	0.000	1	
swit_tl	.8810679	36.992	0.000	1	
rail_age	.8363738	8.817	0.003	1	
swit_age	1.970124	30.454	0.000	1	

/sigma	5.301378				

Test	Restricted	LR statistic	P-Value		
H0:	log likelihood	chi2	Prob > chi2		

lambda = -1	-11465.836	2278.72	0.000		
lambda = 0	-10360.243	67.54	0.000		
lambda = 1	-10807.235	961.52	0.000		

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All coefficients are significant at the 1 percent level (except some of the track district dummy variables). Our a priori expectations of the signs of these variables are given in table 1, section 3 and all estimated coefficients fulfil expectations. There are positive relationships between maintenance costs and output levels, track section length, switches, rail and switch age, joints, quality class and station areas. Conversely, costs are negatively related to the length-distance ratio. These findings are in line with what has previously been found in Andersson (2006).

The estimate of λ , the transformation parameter, is 0.17 and significantly different from zero at the 1 percent level. Hence, we reject the logarithmic transformation of our dependent and transformed independent variables.

Table 3 summarises the estimated Box-Cox elasticities, evaluated at the sample means for output and maintenance costs using expression (5). Standard errors are adjusted using a cluster indicator for track sections, i.e. independence is assumed between track sections, but not within. A challenging result is that the mean cost elasticity with respect to passenger traffic volumes is more than three times higher than the equivalent elasticity for freight. The confidence intervals are not overlapping, indicating a significant difference at the 5 percent level. In other words, passenger trains seem to drive maintenance costs more than freight trains, which is not in accordance with conventional wisdom among track engineers. *Ceteris paribus*, a freight train is considered to do more damage to the track than a passenger train (Öberg et al., 2007).

Table 3. Cost elasticities – Box-Cox

Elasticity	Observations	Mean	Std. Error*	[95% Conf. Interval]	
Freight	648	0.052264	0.001134	0.050026	0.054503
Passenger	648	0.179364	0.003643	0.172443	0.186285

* Cluster adjusted

Figures 1 and 2 contain plots of elasticities from the Box-Cox model. We find increasing elasticities with output, but at a decreasing rate (figures 1 and 2). This shape has also been found in previous work by Andersson (2007a) on Swedish railway maintenance costs and by Link (2006) on German motorway renewal costs.

The estimated elasticities from specification (6) give us reason to reconsider our model and also consider interaction variables, variables that will capture the joint effect from two

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variables. Introducing interaction variables though, has no significant impact on the results in table 2 and 3.

Figure 1. Cost elasticity w r t freight volumes – Box-Cox

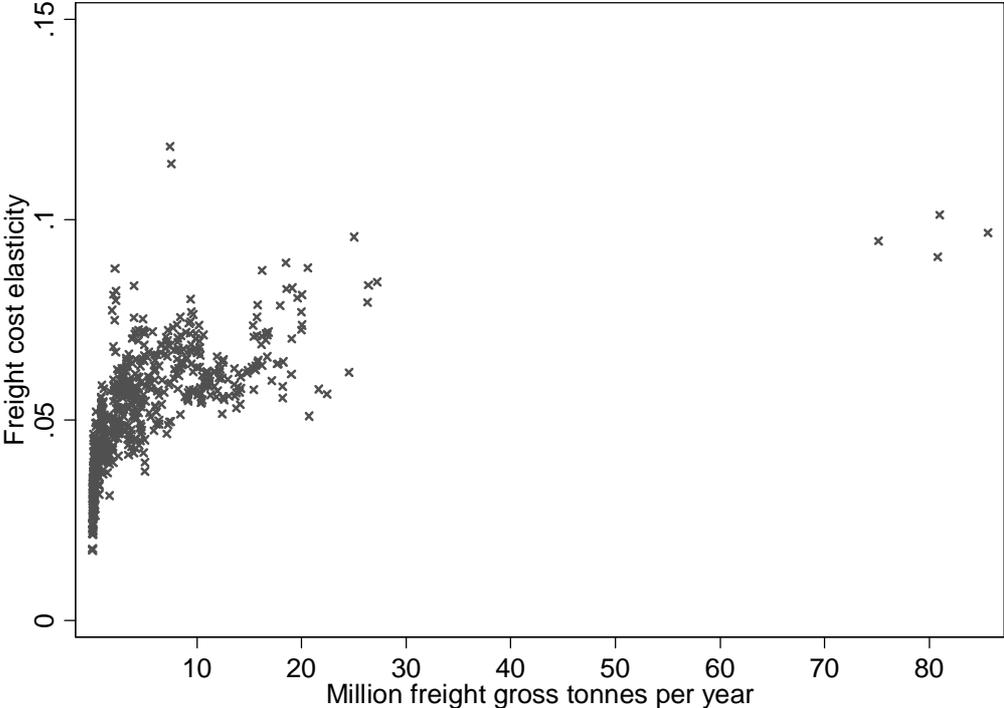
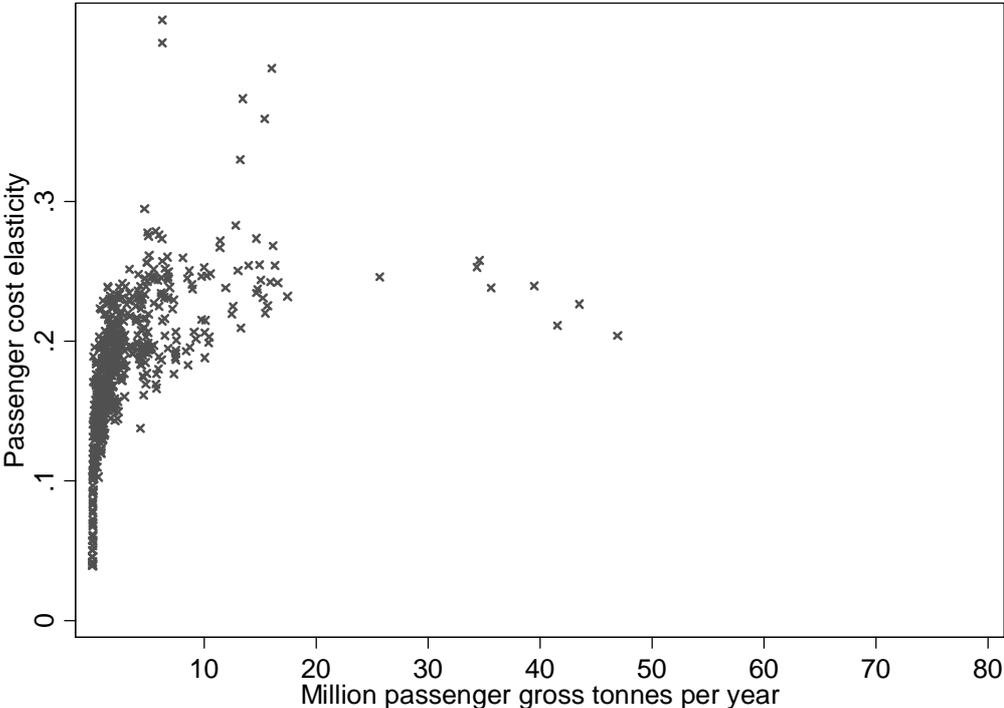


Figure 2. Cost elasticity w r t passenger volumes – Box-Cox



5.2 Dedicated freight lines

In line with the analysis of mixed lines, we have initially estimated a Box-Cox model, but the likelihood ratio test has not rejected the transformation parameter λ being zero. We therefore specify a log-linear model for dedicated freight lines. This model is built on 101 observations and some of the variables used for mixed lines are excluded. Switches, age variables, quality class and joints have proven insignificant, but we use rail weight (*lnrlwgh*) as a quality proxy instead. We also include a squared term for output to capture a non-linear relationship. The final model specification is given in (7).

$$\ln C_{it} = \alpha + \beta_1 \ln fgt_{it} + \beta_2 (\ln fgt_{it})^2 + \beta_3 \ln ld_ratio_{it} + \beta_4 \ln bis_tsl_{it} + \beta_1 \ln rlwgh_{it} + \beta_2 (\ln rlwgh_{it})^2 + \sum_{n=1}^3 \gamma_n year_{it} + \varepsilon_{it} \quad (7)$$

The estimated model is given in table 4 (dummy variables excluded). The signs of the coefficients are in line with our a priori expectations except for length-to-distance ratio, which is now positive. This indicates that costs increase rather than decrease with more meeting points and double tracks.

Table 4. Log-linear regression model estimates – Dedicated freight lines

Linear regression						
					Number of obs =	101
					F(9, 91) =	52.26
					Prob > F =	0.0000
					R-squared =	0.8112
					Root MSE =	.56534

lncm	Coef.	Robust Std. Err.	t	P> t	[95% Conf. Interval]	

lnfgt	1.919855	.6047601	3.17	0.002	.718573	3.121136
lnfgt2	-.0580122	.0237509	-2.44	0.017	-.1051905	-.010834
lnldrat	1.07993	.1948566	5.54	0.000	.6928717	1.466989
lntsl	.7595336	.0851649	8.92	0.000	.5903641	.9287031
lnrlwgh	41.21475	19.12571	2.15	0.034	3.223882	79.20561
lnrlwgh2	-5.527185	2.539759	-2.18	0.032	-10.5721	-.4822659
_cons	-85.22377	34.40323	-2.48	0.015	-153.5616	-16.88598

Table 5 summarises the estimated cost elasticity, evaluated at the output mean using expression (8).

$$\frac{\partial \ln C}{\partial \ln fgt} = \hat{\beta}_{\ln fgt} + 2 \cdot \hat{\beta}_{(\ln fgt)^2} \cdot \text{mean}(\ln fgt) = \hat{\phi}^{LL} . \tag{8}$$

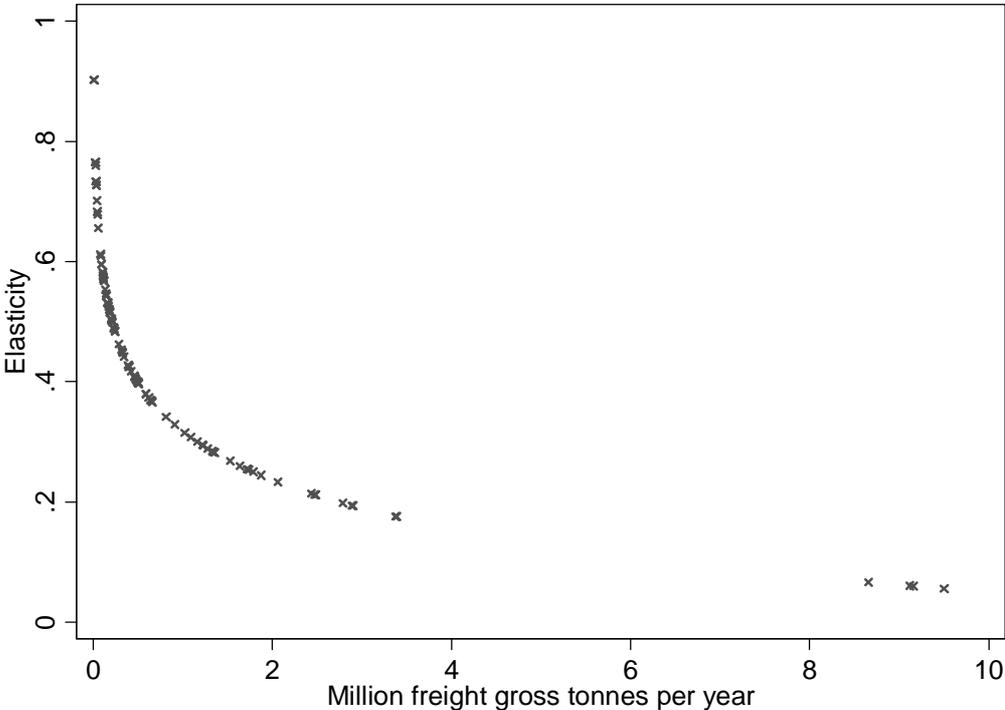
Table 5. Cost elasticity – Dedicated freight lines

Elasticity	Observations	Mean	Std. Error*	[95% Conf. Interval]
Freight	101	0.435141	0.036208	0.361194 0.509088

* Cluster adjusted

The estimate is substantially higher than the freight elasticity in the Box-Cox model. Figure 3 gives a plot of the elasticity function and it is downward sloping as opposed to upward for the mixed line elasticities.

Figure 3. Cost elasticity w r t freight volumes – Dedicated freight lines



5.3 Average and marginal cost estimates

The elasticities derived in sections 5.1 and 5.2 are important inputs in the calculation of marginal costs. The cost elasticities of output are expressed per gross tonne (q), but from a pricing perspective, we also prefer the marginal cost to be distance related and expressed in terms of gross tonne kilometres (q^{gk}). Following Johansson and Nilsson (2004), for output k we express the marginal maintenance cost (9) as

$$MC_k = \frac{\partial C}{\partial q_k^{gtk}} = \frac{\partial \ln C}{\partial \ln q_k^{gtk}} \cdot \frac{C}{q_k^{gtk}} = \frac{\partial \ln C}{\partial \ln q_k} \cdot \frac{C}{q_k^{gtk}} = \phi_{it} \cdot \frac{C}{q_k^{gtk}}. \quad (9)$$

Marginal cost is the product of the cost elasticity ϕ and average cost. By this, we assume that the cost is unaffected by line length at the margin. Estimates of track section marginal costs can be derived by using the output (k) specific elasticity estimates and predicted costs as in (10)

$$MC_{kit}^j = \hat{\phi}_{kit}^j \cdot \frac{\hat{C}_{it}^j}{q_{kit}^{gtk}}, \quad (10)$$

where j indicates mixed or dedicated lines. The calculated marginal costs from (10) are observation specific. In order to adjust for the variation of marginal costs over track sections, we can calculate a weighted average marginal cost. We use the output of each traffic category as a track section weight in relation to total output per category. Estimates of marginal costs from track sections with high traffic levels are given a higher weight than marginal costs from track sections with less traffic.

$$WMC_k^j = \sum_{kit} \left[MC_{kit}^j \cdot \frac{q_{kit}^{km}}{\sum_{kit} q_{kit}^{km}} \right] \quad (11)$$

This allows the infrastructure manager to use a unit rate for wear-and-tear over the network, and still be revenue neutral to using track section specific marginal costs.

Table 6. Average costs

Average cost	Observations	Mean	Std. Error*	[95% Conf. Interval]	
Mixed freight	648	0.682289	0.269658	0.150024	1.214554
Mixed passenger	648	5.609661	2.011954	1.638362	9.580960
Dedicated freight	101	0.224562	0.035756	0.151540	0.297585

* Cluster adjusted

The predicted average maintenance cost (AC) is given in table 6. AC is defined as predicted maintenance cost divided by the output specific gross tonne kilometres. The average maintenance cost per gross tonne km for mixed lines is approximately SEK 0.68 for freight and SEK 5.60 for passenger, while for dedicated lines it is SEK 0.22.

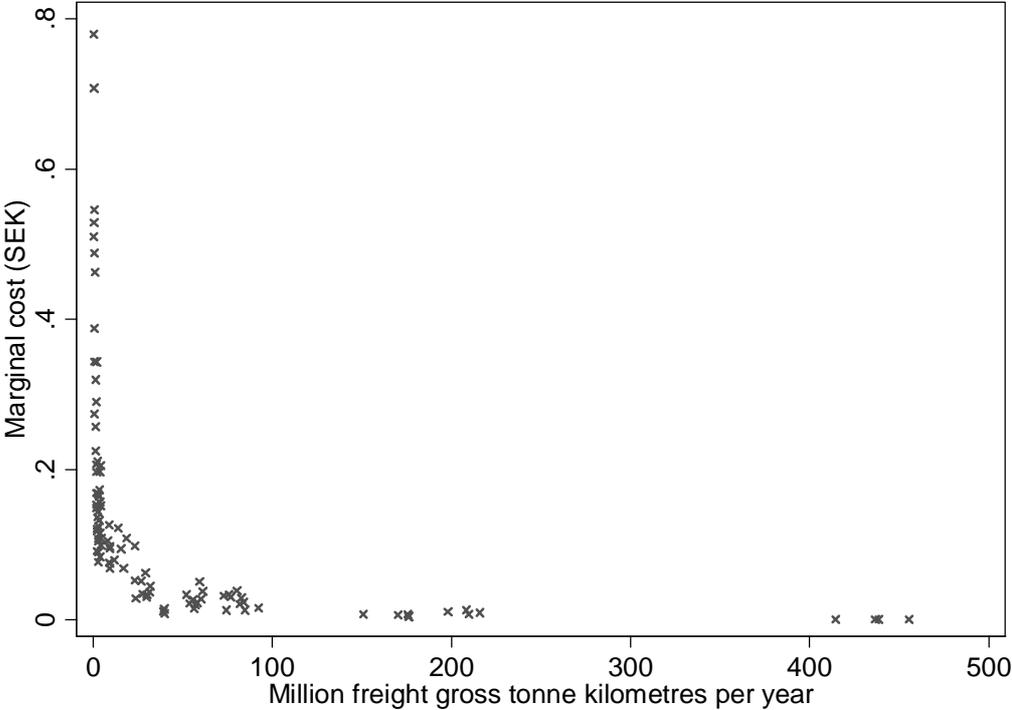
The estimated marginal costs are given in table 7. Mean marginal cost for dedicated lines is SEK 0.126. An output-weighted mean estimate is SEK 0.0168. The marginal cost for freight trains in the Box-Cox model (6) is SEK 0.021 and SEK 0.0014 as a weighted estimate. For passenger trains, the equivalent estimates are SEK 0.296 and SEK 0.0108. We observe some high marginal costs in all three cases for low volume track sections, which drive up the mean values. The marginal costs for dedicated freight lines are plotted in figure 4 and for mixed lines in figures 5-6⁴.

Table 7. Marginal costs

Marginal cost	Observations	Mean	Std. Error*	[95% Conf. Interval]	
Mixed freight	648	0.020780	0.007640	0.005701	0.035860
Mixed freight**	648	0.001425	0.000089	0.001249	0.001600
Mixed passenger	648	0.296449	0.088197	0.122362	0.470536
Mixed passenger**	648	0.010771	0.000714	0.009362	0.012180
Dedicated freight	101	0.126460	0.028038	0.069200	0.183720
Dedicated freight**	101	0.016804	0.002476	0.011747	0.021860

* Cluster adjusted; ** Weighted estimate

Figure 4. Marginal costs - Dedicated freight lines



⁴ We restrict the plot to marginal costs below 1 SEK/Gross tonne kilometre.

Figure 5. Marginal costs - Freight trains - Mixed lines

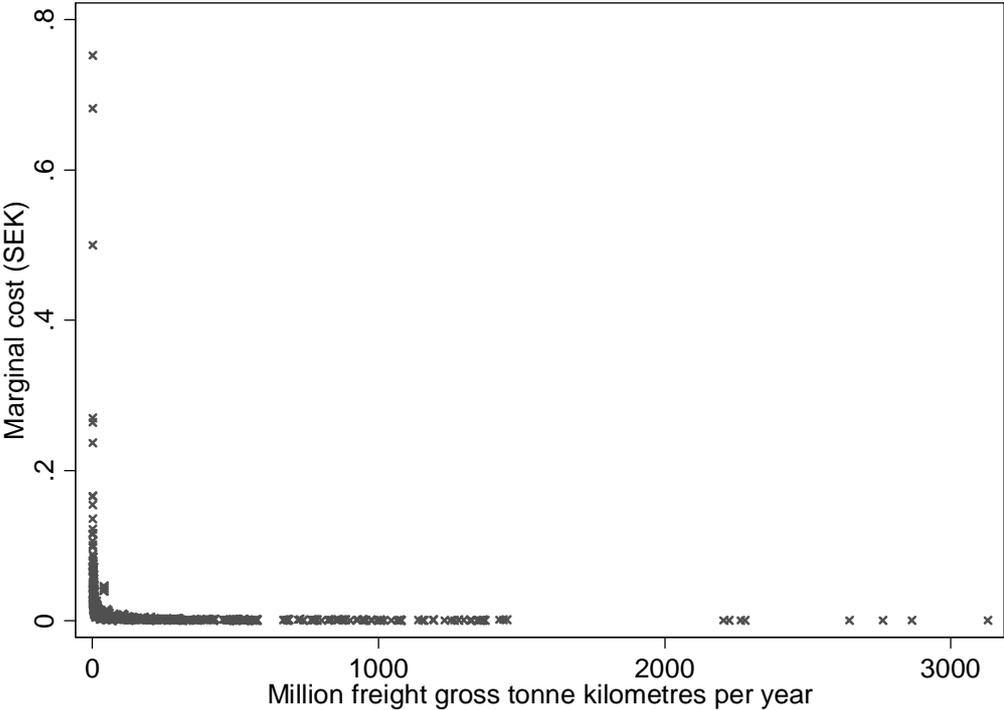
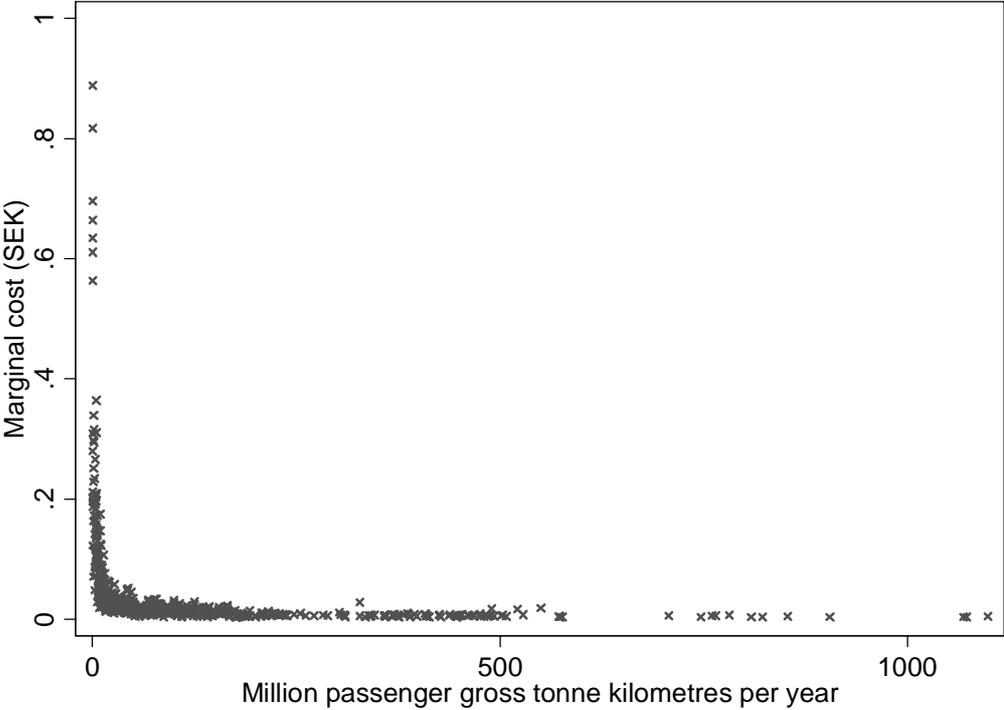


Figure 6. Marginal costs - Passenger trains - Mixed lines



6 Discussion and conclusions

There has been increasing European attention to the issue of marginal costs of railway infrastructure wear and tear in the last decade. The EU-project CATRIN (Cost Allocation of TRansport INfrastructure cost) supports the European Transport Policy, specifically to assist in the implementation of transport pricing for all modes of transport. In this paper, we have analysed maintenance cost data for Swedish railway infrastructure in relation to traffic volumes and other characteristics, and separated the cost impact from passenger and freight trains. Furthermore, we have analysed the choice between logarithmic and Box-Cox regression models and finally checked for differences between railway lines with a mixed passenger and freight traffic pattern and lines dedicated to freight traffic only.

The analysis shows that a Box-Cox regression model is preferred for lines with mixed traffic, but the log-linear model is not rejected for dedicated freight lines.

We observe that most coefficients follow our a priori expectations in terms of cost drivers. One feature though is that the sign of the coefficient for length-distance ratio variable goes from negative (mixed lines) to positive (dedicated freight lines). This seems a little confusing at first glance as a higher ratio indicates higher track time availability. There is a probable explanation though. The dedicated freight lines have fairly low traffic levels, which mean that there is no direct benefit in having multiple tracks with regards to available track time. Hence, track time for maintenance is no scarcity on low-volume lines, but on heavily utilised tracks and adding more tracks to a low-volume line will generate costs. Adding more tracks to a high-volume track on the other hand will reduce maintenance costs as track availability is increased with lower costs as a bonus (less time is spent establishing, re-establishing and waiting during a maintenance activity).

The most challenging result is the ratio between the passenger and freight elasticities in the mixed line case. A suggested explanation to the high passenger elasticity is to look at track management behaviour and rules. Passenger trains operate at higher speeds and require a high-quality track with tougher intervention levels compared to freight trains. This implies more frequent maintenance activities on a mixed line than on a line dedicated to freight only. Management documents at Banverket corroborate this view. Inspection class is a function of speed and gross tonnes (Banverket, 2000). Tamping levels are a function of comfort classes, which are based on quality classes. Higher speeds generate lower tolerance levels in these

quality classes (Banverket, 1997). The cost elasticity is then not solely based on physical wear and tear, but on a combination of wear and tear, and ride comfort. Maintenance policies and actions are highly passenger train service orientated in Sweden and this is reflected in the cost structure.

Still, it can be a matter of omitted variable bias, a common problem in regression analysis. Previous work by Andersson (2007a and 2008) has used fixed effect (FE) estimation on the same data set, using an aggregate output of freight and passenger train volumes. FE estimation solves the omitted variable bias problem if track specific characteristics are time-invariant (Wooldridge, 2002). We are not aware of any FE applications in a Box-Cox framework, but this would be one way of extending this research. Another extension is along the line of acquiring more data, inter alia speeds and axle loads, which are currently not available to us. These variables are used in the deterioration models by Öberg et al. (2007), which allocate freight and passenger train damage to the track.

There is also a difference between the elasticity found for freight trains on dedicated lines and what has previously been found. A 10 percent change in freight traffic on a dedicated line would change maintenance costs by 4.4 percent. The magnitude of the elasticities in previous models (Andersson, 2006, 2007a and 2008), where an aggregate measure of traffic is used, i.e. a total of freight and passenger trains, have been in the range of 0.2 - 0.3. An explanation can be that we have a track that is set up more in line with its usage and costs can therefore be more related to the traffic than when we look at the entire network and use an aggregate output measure. Furthermore, elasticities are falling with output as opposed to the increasing shape found in the mixed line case. The dedicated freight lines differ from mixed lines in terms of tonnage levels and maintenance strategies, and it is therefore difficult to expect identical relationships for both mixed and dedicated lines. The low volumes subsequently lead to higher weighted marginal costs on dedicated freight lines.

The freight elasticity in the model for mixed lines is well below, while the passenger elasticity is more in line with, previous estimates. Marginal costs though differ from what we have previously considered as relevant (Andersson, 2007a and 2008), namely SEK 0.006 – 0.007 per gross tonne kilometre using total gross tonnes as output and panel data estimators. Freight marginal costs are much below this level and also lower than what is currently charged for

wear and tear. Conversely, passenger marginal costs are almost twice of what is previously found and four times the current charge.

A change in the pricing scheme in the direction of the results presented in this paper would lead to more revenues, even if all freight related gross tonnes (70 percent of total tonnage) face a lower wear and tear charge. The joint effect would still give a revenue increase of some 50 percent, with passenger trains paying a much larger share than today. This assumes that total demand for passenger services is unaffected by the price increase.

Most econometric models on railway infrastructure costs have used the data available in the specific case. Within the CATRIN project, we have discussed the potential of using engineering knowledge to enrich our econometric specifications. One important factor identified from this process has been to include some vehicle characteristics, which normally are not collected by railway authorities. Due to lack of information, we have not been able to move towards these suggestions, but they have been highlighted in our work with Banverket as areas where future data collection should aim.

A final observation is that Box-Cox models have introduced some new and interesting possibilities regarding differentiation when analysing Swedish railway infrastructure cost data, but also some issues that we need to attend in future research to improve elasticity and marginal cost estimates. Utilising an efficient variable transformation in conjunction with the information available in panel data is a key for future work.

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8 Appendix 1 Categorisation of Swedish Railway Infrastructure Costs

As a part of CATRIN, the categorisation of different activities related to renewing, maintaining and operating the infrastructure has been discussed. Differences in these categories have been a source of discussion related to why results from econometric studies vary between countries.

This text aims at briefly describing how Swedish railway infrastructure activities are categorised and form different cost categories.

8.1 Funding and coding

The Swedish Rail Administration (Banverket) receives an annual grant from the government. This grant consists of a number of areas to be financed. Specific details about the conditions under which this grant can be used are given in a government instruction to Banverket together with the grant for an upcoming budget year. For 2007, the area of *Track Management* was assigned Million SEK 12 670, with 7 150 to Investments, 4 000 to Operation, Maintenance and Traffic Management and 1 520 to Mortgages and Interests. Banverket sets up an internal financial code system to distribute the grant to activities, c.f. to separate renewals from upgrades and new constructions as they all fall under the Investment category.

There are also a number of activity codes linked to the financial codes and any activity code beginning with the letter *B* is related to the area of Track management. Subgroups are given, to cover for the specification in the grant as well as for internal reporting.

Track access charges are reported as revenue, which can be used on the track. This means that what is actually spent on the track each year is access charges plus government grants.

In general, Banverket follow the European standard SS-EN 13306 Maintenance Terminology in their definitions of maintenance and associated activities.

8.2 Infrastructure operation (Övrig anläggningsskötsel)

This group consists of activities undertaken without affecting functional or technical condition of a unit.

The two categories are winter services (B0707) and other track specific costs for operation and maintenance (B0706), where the former is dominating. Other costs are specifically described as “not being related to a given quality to be performed”. Among these are service of various detection systems and cleaning of station areas.

8.3 Maintenance (Underhåll)

Maintenance is divided into corrective and preventive maintenance, where preventive maintenance can be either condition-based or predetermined.

Corrective maintenance is immediate maintenance (B0801) after observed or reported faults, urgent actions after inspection (B0802) or damages (B0803) that occur after sudden and unexpected incidents.

Preventive maintenance is *condition control*, which consists of safety (B0804) and maintenance (B0805) inspections, and other inspections (B0806); *condition-based maintenance*, which is minor replacements (B0809), tamping (B0810), vegetation control (B0813), rail grinding (B0815), ditching and draining (B0818), painting (B0819), neutralisation (B0822), rail and sleeper adjustments (B0823), overhauls and repairs (B0825) and other condition-based maintenance (B0827). *Predetermined maintenance* (B0826) is interval based according to specific rules and standards.

8.4 Renewals (Utbyten)

Renewals are handled slightly different as mentioned above. The activity codes are common with upgrades and new constructions, but after receiving the grant, an internal budget separates renewals from other construction activities. The internal accounts is therefore easily scanned for renewal costs, but it is impossible (without viewing every single invoice) to see the type of renewal undertaken. The activity codes in this case leave no information but in which phase of the project, the costs are accrued.

Because of this, renewals can be anything from minor replacements (even if this also exists as a maintenance activity) to major rail replacements. The distinction becomes a financial issue rather than a clear cut categorisation of activities. Still, despite the grey zone between maintenance and renewal, the impression from contacts with Banverket is that renewal costs are derived from more large scale activities than minor replacements.

9 Appendix 2 Derivation of the cost elasticity in a Box-Cox model

Consider the following general relationship

$$\frac{y^\theta - 1}{\theta} = \beta \left(\frac{x^\lambda - 1}{\lambda} \right) \quad (\text{A2.1})$$

We are looking for the elasticity $\frac{\partial \ln y}{\partial \ln x}$, which according to the chain-rule is

$$\frac{\partial \ln y}{\partial x} \cdot \frac{\partial x}{\partial \ln x} = \frac{\partial \ln y}{\partial x} \cdot \left(\frac{\partial \ln x}{\partial x} \right)^{-1} = \frac{\partial \ln y}{\partial x} \cdot \left(\frac{1}{x} \right)^{-1} = \frac{\partial \ln y}{\partial x} \cdot x \quad (\text{A2.2})$$

Find $\frac{\partial \ln y}{\partial x}$ by first re-writing (1).

$$\begin{aligned} \frac{y^\theta - 1}{\theta} &= \beta \left(\frac{x^\lambda - 1}{\lambda} \right) \Leftrightarrow \\ y^\theta &= \beta \theta \left(\frac{x^\lambda - 1}{\lambda} \right) + 1 \Leftrightarrow \\ \theta \ln y &= \ln \left(\beta \theta \left(\frac{x^\lambda - 1}{\lambda} \right) + 1 \right) \Leftrightarrow \\ \ln y &= \frac{1}{\theta} \ln \left(\beta \theta \left(\frac{x^\lambda - 1}{\lambda} \right) + 1 \right) = \frac{1}{\theta} \ln \left(\beta \theta \left(\frac{x^\lambda}{\lambda} \right) - \frac{\beta \theta}{\lambda} + 1 \right) \end{aligned} \quad (\text{A2.3})$$

Now, take the derivative of $\ln y$ with respect to x ,

$$\frac{\partial \ln y}{\partial x} = \beta \theta x^{\lambda-1} \cdot \frac{1}{\theta} \cdot \frac{1}{\beta \theta \left(\frac{x^\lambda - 1}{\lambda} \right) + 1} = \beta x^{\lambda-1} \cdot \frac{1}{\beta \theta \left(\frac{x^\lambda - 1}{\lambda} \right) + 1} = \quad (\text{A2.4})$$

We want $\frac{\partial \ln y}{\partial x} \cdot \frac{\partial x}{\partial \ln x}$ which is,

$$\beta x^{\lambda-1} \cdot \frac{1}{\beta \theta \left(\frac{x^\lambda - 1}{\lambda} \right) + 1} \cdot x = \beta x^\lambda \cdot \frac{1}{\beta \theta \left(\frac{x^\lambda - 1}{\lambda} \right) + 1} \quad (\text{A2.5})$$

From (A2.3), we can see that the second factor in (A2.5) is $\frac{1}{y^\theta}$, which gives the elasticity as

$$\frac{\partial \ln y}{\partial x} \cdot \frac{\partial x}{\partial \ln x} = \frac{\beta x^\lambda}{y^\theta} \quad (\text{A2.6})$$

or when $\theta = \lambda$,

$$\frac{\partial \ln y}{\partial x} \cdot \frac{\partial x}{\partial \ln x} = \beta \left(\frac{x}{y} \right)^\lambda \quad (\text{A2.7})$$