Multi-unit common value auctions: Theory and experiments
Dedikation
Till Kicki, Lova och Laban
Multi-unit common value auctions: Theory and experiments
Abstract

Research on auctions that involve more than one identical item for sale was almost non-existing in the 90’s, but has since then been getting increasing attention. External incentives for this research have come from the US spectrum sales, the European 3G mobile-phone auctions, and Internet auctions. The policy relevance and the huge amount of money involved in many of them have helped the theory and experimental research advance. But in auctions where values are equal across bidders, common value auctions, that is, when the value depends on some outside parameter, equal to all bidders, the research is still embryonic.

This thesis contributes to the topic with three studies. The first uses a Bayesian game to model a simple multi-unit common value auction, the task being to compare equilibrium strategies and the seller’s revenue from three auction formats; the discriminatory, the uniform and the Vickrey auction. The second study conducts an economic laboratory experiment on basis of the first study. The third study comprises an experiment on the multi-unit common value uniform auction and compares the dynamic and he static environments of this format.

The most salient result in both experiments is that subjects overbid. They are victims of the winner’s curse and bid above the expected value, thus earning a negative profit. There is some learning, but most bidders continue to earn a negative profit also in later rounds. The competitive effect when participating in an auction seems to be stronger than the rationality concerns. In the first experiment, subjects in the Vickrey auction do somewhat better in small groups than subjects in the other auction types and, in the second experiment, subjects in the dynamic auction format perform much better than subjects in the static auction format; but still, they overbid.

Due to this overbidding, the theoretical (but not the behavioral) prediction that the dynamic auction should render more revenue than the static fails in the second experiment. Nonetheless, the higher revenue of the static auction comes at a cost; half of the auctions yield negative profits to the bidders, and the winner’s curse is more severely widespread in this format. Besides, only a minority of the bidders use the equilibrium bidding strategy.

The bottom line is that the choice between the open and sealed-bid formats may be more important than the choice of price mechanism, especially in common value settings.

Keywords: Multi-Unit Auction; Common Value Auction; Laboratory Experiment; Game Theory
Acknowledgement

At the beginning of time, I started as a research assistant at VTI. Time passed, and I enrolled in the PhD program at Örebro University. Time passed again, and again. Finally, quite late in time, I am writing this. There is an end after all!

Along this never-ending journey, I have had the opportunity to meet and discuss my work, and the work of others, with many inspiring and clever colleagues at the Department of Economics of VTI in both Stockholm and Borlänge. This thesis is the offspring of seminars and discussions with all of you. Thanks! * I would especially like to thank my two supervisors Lars Hultkrantz and Jan-Eric Nilsson for sharp comments and straightforward critique. And of course Gunnar Lindberg for his patience during this eon of time.

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During the timespan of this thesis, I engaged in a procreation process with the love of my life, Kicki Fjell, not only once but twice! I am indebted to Kicki, Lova and Laban for being there; you could not do more for me!

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1 Introduction

When governments started to realize that well-designed auctions were most likely a better method to allocate resources than the beauty contest, where bureaucrats had to rely on business plans which made the process susceptible to corruption, the incentives for testing auction theory increased. This is especially true for multi-unit auctions, the category into which many of the new auction markets fall.

But, still, there is a vast knowledge gap in this field, particularly in cases where values are equal across bidders; that is, when the value depends on some outside parameter, common to all bidders. Such auctions are referred to as common value auctions.

Examples of such auctions are (CO$_2$) allowance auctions, electricity auctions and bond auctions. All three can be modeled as common value auctions. In the allowance auction case, the value is a proxy for the social abatement cost, in the electricity auction the value comes from the electricity price, and in the bond case the value is driven by the interest rate.

The complexity of this type of auctions stems from, first, bidders demanding more than one unit and, second, the value being common to all bidders. This makes closed-form equilibrium analysis difficult.

The following sections provide a unifying framework and a summary of the papers of this thesis. Section 2 offers a condensed survey with, first, single-unit auctions and, second, multi-unit auctions, where theory is related to experiments. The section also serves as the theoretical scope of the thesis. Then follows a brief summary of the essays in section 3, and last, section 4, on some lessons learnt, concludes this summary.
2 Auction theory vs. laboratory experiments

2.1 Private value auctions

Even though auctions have been used since antiquity for the sale of a variety of objects, the modern analysis of auctions as games of incomplete information started with the work of William Vickrey (1961). He introduced the independent private value model, where each bidder’s valuation is independent of the information held by her opponents, and derived equilibrium strategies for the first-price auction as well as the open, and the sealed-bid, second-price auctions, when values come from a uniform distribution. The second-price auction, introduced in the same paper, was shown to be an auction form with truthful bidding and efficient outcomes.

Vickrey also recognized that the expected revenues in first- and second-price auctions were the same, particularly under arbitrary distributions. This was formally proved in a subsequent paper, Vickrey (1962). The finding consisted of some special cases of the (today) celebrated Revenue Equivalence Theorem developed, roughly at the same time, by Riley and Samuelson (1981) and Myerson (1981) The theorem states:

Assume each of a given number of risk-neutral potential buyers of an object has a privately known signal independently drawn from a common, strictly increasing, atomless distribution. Then any auction mechanism in which (i) the object always goes to the buyer with the highest signal, and (ii) any bidder with the lowest-feasibly signal expects zero surplus, yields the same expected revenue.

It is a quite remarkable result, that, on average, all auctions with these two properties generate the same revenue to the seller. This result does not only apply to (independent) private value (PV) models, but to more general common value models, provided that the bidders’ signals are independent.

Surprisingly, this result does not hold in the laboratory. Specifically, there should be strategic equivalence between the two most common first-price auctions; the (sealed-bid) first-price auction and the Dutch auction, as well as

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1 In a first-price auction, you pay what you bid (if you win), whereas, in a second-price auction, you pay the second highest bid (if you win).
2 Vickery was awarded the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1996, to a certain extent because of the two mentioned papers.
3 In the Dutch auction, the auctioneer begins with a (sufficiently) high price. The price is gradually reduced until a bidder indicates her interest. The object is then sold to this bidder at the given price.
the two most common second-price auctions: the (sealed-bid) second-price auction, and the English auction. But Coppinger et al. (1980) find that there is no strategic equivalence in first-price and Dutch auctions, and that the Dutch auction yields lower prices. Kagel et al. (1987) report failures in the strategic equivalence of second-price and English auctions, with lower prices in the English auction. The English auction is the only auction in which bidding converges to the dominant strategic prediction. The rationale for this seems to be the transparency of the dynamic, or open, (ascending) auction, and subjects learn how to behave quickly enough.

2.2 Common value auctions

At the other end of the value spectrum, we have the (pure) common value. In the common value (CV) model, all bidders assign the same value to the unit for sale, but bidders have different private information about the (unknown) value at the time that they bid. Thus, even though the ex-post value is common for everyone, it is unknown at the time of bidding. The CV model was first introduced by Wilson (1969). He also developed the first closed-form equilibrium analysis of the winner’s curse in that article. The core of the winner’s curse (WC) is that a bidder must not ignore the (adverse selection) effect inherent in winning the auction, in order not to pay more than the estimated worth of the object. (Even if a bidder’s estimate is an unbiased estimator of the value, the largest of all the estimators is not. The max function is convex and thus overestimates the value.)

The presence of the WC as an empirical fact was pointed out by Capen et al. (1971) in the context of bidding for offshore oil drilling leases. They claimed that the oil companies suffered unexpectedly low returns in oil lease auctions. An oil lease auction is a typical CV auction, since the value of the oil in the ground is essentially the same for all bidders. This type of auction spurred the development of the pure CV model (or the mineral rights model).

A seminal experiment by Bazerman and Samuelson (1983) on the WC showed that, in general, subjects were pretty inclined to overbid, since they overestimated the value and, as a consequence, fell prey to the WC. Subsequent experiments show that the WC is prevalent in many different settings, and that if there is some learning, it is context specific. Even if subjects over time learn to avoid getting a negative profit from the WC in an experimental set-up, this does not carry over to another set-up. For example, in the open

\[4\] In the English auction, the auctioneer starts with a low price and gradually raises it as long as there are at least two interested buyers. The auction stops when only interested buyers remain. This might be the most common auction form used in practice.
variant of the second-price auction, the English auction, subjects with higher estimates of the true value get information of the value as a consequence of lower valued bidders dropping out. This alleviates, but does not eliminate, the WC. However, subjects do not transfer this experience to the static, or sealed-bid, variant of the same auction.

2.3 Interdependent value auctions

The PV and the CV models are extreme cases, at either end of the spectrum of the interdependent value model. The value in this model can, but must not, depend on other bidders’ information, or signals. Milgrom and Weber (1982) developed the symmetric model with interdependent values (IV) and affiliated signals, encompassing both the PV and CV models. Affiliated signals play a part in IV models, meaning that bidders’ signals are correlated, positively or negatively. When the signals are affiliated, the revenue ranking theorem (RET) above ceases to hold, since independent signals are assumed.

Their article derives the general revenue ranking, as well as other equilibrium characterizations, of the three most common auction formats; the first price, the sealed-bid second price, and the open second price, i.e. the English auction. The main results are that the English auction leads to a higher expected revenue than sealed-bid second-price auctions which, in turn, lead to a higher revenue than first-price auctions.

The intuition for this is that, in the open auction with interdependent values, there is a great deal of information revelation as bidders drop out when the price increases. This implies that the price paid will not only depend on private information, but also on all other bidders’ information. The more the price depends on other bidders’ information, the higher will be the price (revenue); that is, the winner’s private information becomes less valuable. (In the second-price auction, the price depends on one other bidder besides the winner, but in the first-price auction the price only depends on the winner.)

However, when the revenue ranking from Milgrom and Weber (1982)’s article is experimentally tested in laboratory settings, focusing on pure CV models only, the conclusion is that, just like the RET, it does not hold either. The main reason seems to be that subjects in the experiments often fall prey to the WC, and therefore earn a negative profit. This happens for both inexperienced bidders and those with varying degrees of experience. The overbidding is alleviated in the English auction, thereby giving the subjects a greater profit,

5 If the signals are positively affiliated, it roughly means that, if a subset of the signals is all large, then this makes it more likely that the remaining signals are also large.
which translates into less revenue. This is contrary to the revenue ranking which implies that the English auction creates a greater revenue. But as the number of bidders increases, the difference in revenue between auction formats decreases. (A comparison between the English and the first-price auction can be found in Levin et al. (1996).)

For the three value structures described above, the subjects generally seem to understand the English auction best in experiments. It is in this auction format that the behavior matches the predicted theory. The rationale is the inherent price discovery mechanism in the English auction, which is absent in sealed-bid formats. This mechanism is important in IV structures, since bidders learn how aggregate demand changes as the auction proceeds. That is, the transparency of the dynamic auction seems to make it relatively easy for subjects to understand that they should not bid above their value, which is the dominating equilibrium in the PV case. (In the PV case, where all subjects know their value, and where this value does not depend on other bidders’ values, the price discovery mechanism has no bearing.)

2.4 Multi-unit auctions

When the US government started selling radio spectrum licenses, and when the design of Internet auctions became important, the theory on multi-unit auctions started to advance. In a multi-unit auction, the implicit meaning is that bidders demand multiple units; nothing drastic happens in the theory when selling multiple units, as long as individuals demand a single unit. The commodities for sale could be either homogenous or complements; the focus here is on homogenous units.

Vickrey (1961) also pioneered in this setting, as he described an efficient mechanism in multi-unit settings in the PV environment, nowadays called the Vickrey auction. Ausubel (2004) then came up with an open format that has the same outcome as the multi-unit Vickrey auction in a PV setting, and also continues to be efficient in an affiliated value environment, in contrast to the static Vickrey auction.

Manelli et al. (2006) experimentally compare the static Vickrey auction with the Ausubel auction, also known as the dynamic Vickrey auction, in both a PV setting and an IV setting, in which the values are affiliated. They conclude that due to overbidding being slightly more frequent in the Vickrey auction, the revenue from the Vickrey auction is greater, while the efficiency is lower in the Ausubel auction. In the IV setting, they observe less overbidding and a trade-off between efficiency and revenue; the Vickrey auction is more efficient while the revenue is higher in the Ausubel auction.
A seminal (game theoretic) article on common value multi-unit auctions is that by Wilson (1979). He found that, in an auction of shares, there existed equilibria with prices lower than if the item were sold as an indivisible unit. Later, Ausubel and Crampton (2002) showed that the efficiency of the second-price, multi-unit auction might break down due to demand reduction in an IV model. Demand reduction, which is the phenomenon that bidders reduce demand (on marginal units) in favor of a lower market-clearing price, has since then been shown in several experiments in PV, IV and CV models.
3 Essays in the thesis

Three self-contained and self-authored essays, which comprise the thesis, are summarized below and related in the following way. In essay I, a multi-unit CV model is constructed, and three different sealed-bid auction mechanisms are analyzed. In essay II, the model from paper I is scrutinized through a laboratory experiment. All hypotheses tested are derived from the analysis of essay I. Essay III also conducts a laboratory experiment, but the analysis distinguishes between the open and the sealed-bid uniform auction mechanisms. In this experiment, many of the hypotheses also come from the analysis in essay I; but also from behaviors and anomalies in the first experiment, essay II.

3.1 Essay I - Analysis of discrete multi-unit, common value auctions: A study of three sealed-bid mechanisms

In this paper, we suggest and evaluate a simplified common value model, which is also discrete in both values and bidding. The common value is generated as the sum of two integers, where one integer is independently displayed for each bidder and, first, serves as a signal for the bidder and, second, operates as the bidder’s type in the Bayesian game. Both the value and the signals are thus affiliated. The model has a two-unit demand environment, and the number of bidders is two, three or four. Three formats are considered, the discriminatory, the uniform, and the Vickrey auction.

Since the value of each of the two items is the same for every player, there is no efficiency issue; the relevant task for the study is rather to identify equilibrium strategies and compare the revenues of the three auction formats. In the uniform and the Vickrey auctions, there are no unique strategies, but if bidders use the payoff maximizing strategies we find the (theoretical) prediction that, of the three auction formats with two players, the discriminatory auction gives the highest revenue to the seller, and that the uniform and the Vickrey auction both give zero revenue. The zero revenue equilibria are a consequence of extreme demand reduction. Nonetheless, considering the latter two, the Vickrey auction may be placed above the uniform auction in revenue ranking because of the existence of the demand revealing equilibrium in the Vickrey auction. We also find that, in equilibrium, bidders bid the same amount on both items in the discriminatory auction; a phenomenon we do not notice in either of the other auction formats.

Using these results, we make a contribution to the ongoing discussion of which of the two formats, the uniform or the discriminatory, gives the most revenue.
The paper suggests that the discriminatory auction is a better alternative than the uniform auction, especially when there are only two bidders. One reason is the potentially extreme demand reduction in the uniform auction. Even with a slight demand reduction, though, the uniform auction gives less revenue in our model than the discriminatory auction.

3.2 Essay II - Multi-unit common value auctions: A laboratory experiment with three sealed-bid mechanisms

This study features a discrete, in the sense that the value of the unit and bidding are only allowed in integer numbers, common value auction with independent (one-dimensional) private signals, where the seller offers two identical units and the buyers demand both. All three auction formats are tested and subjected to a variation in the number of bidders; 2, 3 or 4 buyer-groups are employed, as are repeating bid rounds (15 - 20 rounds) for each subject. Five main questions are scrutinized. (i) which auction format gives the greatest revenue?; (ii) how does the number of bidders affect revenue?; (iii) is there demand reduction in the uniform and Vickrey auctions?; (iv) what are the implications of repeating the auction several rounds for the subjects, do we see any learning effects?; and (v) is there a winner’s curse; that is, do bidders ignore the informational content inherent in winning, and bid too high? For the first four questions, we have hypotheses derived from the predictions in essay I; for the fifth question there is a behavioral hypothesis of the winner’s curse since the winner’s curse not arise in optimum play, which we search for in essay I.

Starting with revenue, we find that the Vickrey auction always gives the least revenue, regardless of group size. The uniform and the discriminatory auctions run a close race, but due to the non-expected result in 2-player groups, the uniform auction is weakly better. (The hypothesis for the uniform auction was that, in 2-player groups, the subjects would play more according to the demand reduction theory. But, generally, they did not.) Still, for larger group sizes, the difference between them is small. The answer to the second question is without doubt in this setting; the more bidders in an auction, the larger the seller’s revenue. All formats support this statement. Third, we see demand reduction, but we hardly see an extreme demand reduction, that is, zero bidding on the second unit. Fourth, we find that subjects do learn to play equilibrium strategies in the course of the play, at least in the discriminatory auction. Moreover, they continue to learn until the final rounds.

For the last question, we find that the winner’s curse is highly present, mostly in the uniform and discriminatory auctions, but also in the Vickrey auction. We distinguish between bidding above the naive, conventional expected value
and between the conditioning expected value of winning up to the naive expected value, and find that it is twice as common to bid in this first interval. This indicates that subjects have a problem understanding the winner’s curse.

From the results, we first notice that as the number of players increases, the pricing rules converge in collecting revenue. When there were only two bidders in the auction, all formats were significantly different in revenue raising; but when there were four bidders, the difference became insignificant. Thus, attracting bidders, or ensuring competition, could be much more important than the selected auction form.

There was especially one odd result in the experiment, namely the high revenue for 2-player groups in the uniform auction. It was rather unexpected because of the anticipated low revenue equilibria outcome in this group. One possible explanation is the competitive element; subjects did not play the theoretical equilibrium at all; they wanted to win the object(s), no matter the costs. Holt and Sherman (1994) explain this as the joy of winning phenomenon in their study. In the present study, it was not only encountered in this particular group size, but was also pretty common in all group sizes in all auction formats.

3.3 Essay III - Multi-unit common value auctions: An experimental comparison between the static and the dynamic uniform auction

It is still an open question whether the open or static format should be used in multi-unit settings, in a uniform price auction. In the private value case, a couple of field and laboratory experiments have shown that one must be cautious about using open formats.

The present study conducts an economic experiment in a common value environment, and both the static and dynamic formats are used in two group sizes: 3 and 6-player groups. In letting the larger groups’ configurations (in own demand) be exactly two times the smaller groups, and letting the supply be equal in both groups, it effectively is as comparing a loose and a tight cap at the same time (if bidding does not adapt to the increasing number of bidders). The loose cap, represented by the 3-player groups, has the relation \( \frac{S}{D} \) of supply (numerator) and aggregated demand (denominator), whereas the tight cap, or the 6-player groups, has the relation \( \frac{S}{D} = \frac{1}{2} \). Moreover, the two group sizes always have the relation \( \frac{1}{2} \) between the large demander (numerator) and the small demander (denominator). The tight cap resembles the European Union Emission Trading Scheme auctions conducted in Great Britain (but open for participants throughout the EU).

The main results from the experiments are;
Subjects’ bids do not decrease in response to an increased number of bidders, contrary to the predictions of theory. Since the bids do not increase either, we conclude that doubling the number of players is equivalent to halving the supply.

Seller revenue is significantly greater in the sealed-bid format. But it comes at a cost in terms of a considerably more negative profit for buyers, and nearly half of the auctions ended with a negative profit for the subjects.

In line with this is the considerably less WC in the open format, both bidding in the winner’s curse interval (see essay II above) and experiencing a negative profit. There is also a notable number of bids above the conventional, naive, expected value, especially in the static format.

The more bidders (or, the tighter the market), the greater the revenue.

None of the formats seem to result in high bids that coincide with individual rationality. Subjects in the open format do perform somewhat better, but not well enough since just 1/5 of all subjects’ first unit bid/dropout is at, or below, the expected value of the unit in the dynamic format.

The demand reduction, measured as the bid spread, is significantly lower in the dynamic format.

Thus, we conclude that in deciding which of the two auction formats of the uniform price auction that is preferred in CV environments, we have to determine if (i) collecting the most revenue or (ii) avoiding the most negative bidder profit is the most important criterion in the choice process. The dynamic auction seems to be a better choice, especially if players are without experience. It facilitates price discovery, thereby alleviating the overly aggressive bidding.

The bottom line is that the choice between the open and sealed-bid formats may be more important than the choice of price mechanism, especially in CV settings.
4 Lessons learned, policy implications and future research

Many of the problems discussed in section 2 between theory and experiment, and encountered in a single-unit environment, are also detected in experiments in the multi-unit settings of this thesis. That is, all theoretical predictions from the first essay were not fulfilled in the second essay. And, likewise, not all hypotheses in the third essay were confirmed in the experiment. For example, there was a great deal of overbidding in both experiments and in all auction formats.

The reason for the overbidding is not easy to pinpoint. One rationale is the complexity of the game at hand; the concept of common value is not intuitive for most subjects, or so it seems. Both experiments used inexperienced players. Even though they had several dry-rounds before the experiment began, and had time to learn during these rounds, they had probably never been faced with some of the auction types. But the phenomenon of being a victim of the WC is not unique to inexperienced bidders, Kagel and Levin (2002) have shown that overbidding is a robust feature, also prevalent among professionals. This is consistent with the results in the two experiments in the thesis, i.e. that subjects continue to suffer from overbidding in later rounds. The joy of winning is another rationale; the competitive effect takes over the rationality.

Even though the latter two essays differ in their laboratory set-up; for example, in the number of bidders, the demand per bidder, how the value is generated, open and sealed-bid formats etc, the overbidding is constantly there. In the last essay, subjects in the open uniform auction were less prone to overbid as compared to the sealed-bid counterpart, which is best rationalized through the price discovery mechanism this format possesses, inherent from the transparency of the same.

Thus, in conducting a multi-unit CV auction (procurement), the auctioneer should, first, attract potential bidders to participate; the number of bidders may be the most important variable when it comes to enhancing revenue. Second, given the number of bidders, the auction designer should take into consideration the choice between the open and sealed-bid formats, the latter producing more revenue, and the former reducing the bidding errors. Reducing bidding errors is paramount in the long run since buyers are not likely to come back if they lose money; hence the two cases mentioned here are intertwined.

Thus, the dynamic uniform auction is a much preferred mechanism over the static counterpart when it comes to many multi-unit auctions. For example, when selling CO\textsubscript{2} allowances, the government must look after the industry as a whole, or when selling bonds, take careful account of the welfare in the nation. Here, the dynamic auction offers a solution by means of cushioning
the overbidding and helps find the right price for the units for sale. Moreover, concerning procurement, the more risk there is in a sealed-bid tender, the more the final contract tends to be at cost-plus pricing. Thus, since risk is a CV, there could potentially be huge efficiency gains by more often using the dynamic auction in risky procurements.

There is a need for much more theoretical and experimental research on multi-unit auctions in general and CV auctions in particular. Since the WC is by now well known and anticipated in experiments, the next step should be to design experiments in such a way that the overbidding is neutralized; that is, find a way of abstracting the joy of winning (or whatever it may be called) from the bids.
References

Analysis of discrete multi-unit, common value auctions: A study of three sealed-bid mechanisms

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Abstract
This paper proposes a discrete bidding model for both quantities and pricing. It has a two-unit demand environment where subjects bid for contracts with an unknown redemption value, common to all bidders. Prior to bidding, the bidders receive private signals of information on the (common) value. The value and the signals are drawn from a known discrete affiliated joint distribution. The relevant task for the paper is to compare the equilibrium strategies and the seller's revenue of three auction formats. We find that, of the three auction formats below with two players, the discriminatory auction always gives the largest revenue to the seller; both the uniform and the Vickrey auction have zero revenue equilibrium strategies that put them further down in the revenue ranking. In equilibrium, bidders bid the same amount on both items in the discriminatory auction; a phenomenon not noted in either of the other auction formats.

Keywords: Laboratory Experiment; Multi-Unit Auction; Common Value Auction

JEL codes: C91; C72; D44

1 Introduction
The research on auctions that involve more than one identical item for sale has recently been getting increasing attention, which is not strange considering the huge amount of money involved in many multi-unit auctions. Auctions for treasury bills, spectrum rights, procurement, and emission permits are just a few examples where more than one identical item is sold at the same time for billions of dollars.

An analytical problem, so far, when these sales are scrutinized is the lack of closed form solutions for the equilibrium bidding strategies. Not only do we
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1 Introduction

The research on auctions that involve more than one identical item for sale has recently been getting increasing attention, which is not strange considering the huge amount of money involved in many multi-unit auctions. Auctions for treasury bills, spectrum rights, procurement, and emission permits are just a few examples where more than one identical item is sold at the same time for billions of dollars.

An analytical problem, so far, when these sales are scrutinized is the lack of closed form solutions for the equilibrium bidding strategies. Not only do we
face the problem with multiple units, we also have units that may have the same (common) value for all bidders. For example, in the treasury auction, the value is driven by the interest rate, which is common to all bidders, while in emission permit auctions, the value is a proxy for the social abatement cost.

In many settings, the continuous bidding paradigm is also in doubt, since many auctions are conducted through discrete bids. This is especially true in some Internet auctions. For example at Tradera, a Swedish-based Internet site, the bid-increments for items depend on their value. For values between SEK 1 (approx. $0.15, €0.11) and SEK 10000 (approx. $1500, €1075), the increment could be as large as 100 percent of the value down to 1 percent of the value. (It oscillates between these values because different sections of values have different increments, see www.tradera.com.)

In this paper, we analyze a discrete (in both values and bidding) common value model. To focus the attention, the model has a two-unit demand environment where subjects bid for contracts with an unknown redemption value, common to all bidders. Prior to bidding, the bidders receive private signals of information on the common value. Both the value and the signals are drawn from a known discrete affiliated joint distribution.

The specific set-up used in this paper emanates from an unpublished wind-tunnel experiment by Lindén et al. (1996). There, the idea was to find a different way of representing the common value and, especially, make it easier for the subjects to understand the winner’s curse problem inherent in common value auctions. In the experiment of Lindén, three dice were rolled in each round and the common value was generated as the sum of the three dice. One of the dice was then shown independently to each player. In this way, a common value environment with independent private signals was created. The main objective was to compare the revenues of the uniform auction, the discriminatory auction and the Vickrey auction in a two-unit demand environment.

In the present model, a Bayesian game is constructed for the analysis. The common value is generated as the sum of two integers, where one integer is independently displayed for each bidder and, first, serves as a signal for the bidder and, second, operates as the bidder’s type in the game. Three auction formats are considered, the discriminatory, the uniform, and the Vickrey auction.

Even though the model is simple, with only two units for sale, it may help give

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1 The winner’s curse is defined as the failure to understand that the announcement of winning leads to bad news, if not accounted for when bidding. That is, the possibility that, upon winning, one pays more than the estimated worth of the object.
some insights for auctions with many more objects for sale, such as bond or emission permit auctions, which are both common value auctions. Riksgälden (the Swedish National Debt Office) has recently communicated that it will stick to the discriminatory auction, Riksgälden (2007). It also states that it is harder to reject the Vickrey auction, but since this mechanism is not used anywhere in the world, presumably due to its complicated nature, a switch to it cannot be recommended.

Since the value of each of the two items is the same for every player, there is no efficiency issue; the relevant task for the study is rather to identify equilibrium strategies and compare the revenues of the three auction formats. In the uniform and the Vickrey auction, there are no unique equilibrium strategies, but if bidders use the payoff maximizing strategies we find that, of the three auction formats with two players, the discriminatory auction gives the highest revenue to the seller, and that the uniform and the Vickrey auction both give zero revenue. The zero revenue equilibrium is a consequence of extreme demand reduction. We also find that, in equilibrium, bidders bid the same amount on both items in the discriminatory auction; a phenomenon we do not note in either of the other auction formats.

Our results make a contribution to the ongoing discussion of which of the two formats, the uniform or the discriminatory, gives more revenue, suggesting that the discriminatory auction is a better alternative than the uniform auction, especially when there are only two bidders. One reason is the potential extreme demand reduction in the uniform auction. Still, even with a slight demand reduction, the uniform auction gives less revenue in our model than the discriminatory auction. Related to the first paragraph above, we also see that many government securities auctions worldwide still use the discriminatory mechanism.

This research is mainly related to the literature of multi-unit demand auctions with interdependent values. There is little earlier theoretical work on multi-unit demand, common value auctions against which to directly compare our results, except for the theoretical article from Álvares and Mazón (2010). They have a theoretical model similar to this in a continuous setting. Like the present paper, they find that the discriminatory auction has an equilibrium where bidders bid the same price for both units, whereas the uniform auction does not. Moreover, they show that the comparison of the seller’s expected revenue across auction formats only depends on the ratio of the precision of private information to the precision of public information.

The earlier work on optimal auctions and revenue comparison by Vickrey (1961), Myerson (1981) and Riley and Samuelson (1981), which focused on independent private value settings, and the seminal paper on interdependent values and affiliated signals by Milgrom and Weber (1982), as well as the first common value paper by Wilson (1969), generally assume that the buyers just demand one unit each.
Ausubel and Crampton (2002) generalize earlier multi-unit demand models by allowing each individual to demand an arbitrary number of units and by allowing the valuations to be correlated. They show that demand reduction is prevalent in the uniform auction and that, in many cases, the discriminatory auction outperforms the uniform-price auction, even though they also show that the revenue ranking of the three formats (uniform, discriminatory and Vickrey) is ambiguous. Moreover, since there is a greater chance that the stronger bidders’ bids become the market clearing price than the weaker bidders’ bids, strong bidders shade their bids relatively more than weak bidders. This, in turn, may cause the stronger bidders to lose units to otherwise weaker bidders.

In their analysis, they assume an infinitely divisible good rather than discrete goods to simplify the calculations, an approach proposed by Wilson (1979). Wilson showed, by means of examples, that the uniform auction is unfavorable for the seller in terms of revenue when compared to the discriminatory auction since it had collusive equilibria. Two other papers that also use the divisible-good are Back and Zender (1993) and Wang and Zender (2002) which show that the discriminatory auction yields a unique equilibrium with greater expected revenues than the uniform auction. They also make a point of the fact that insights into their single unit framework cannot be directly replicated in their multi-unit counterpart. Thus, all these papers are in line with the present paper.

For discrete units, there are two papers by Engelbrecht-Wiggans and Kahn, both dealing with the independent private value case. Engelbrecht-Wiggans and Kahn (1998a) consider a uniform price auction of two items and find that, in equilibrium, there is a positive probability of demand reduction, manifested as a bid-shading for each bidder’s lower-value items in such a way that the bids are strictly below their valuation for these items. This can also be seen in the interdependent value model in our study. Engelbrecht-Wiggans and Kahn (1998b) present a similar model but with a discriminatory format where \( M \geq 2 \) items are for sale and each bidder has a demand for two items. They establish that there is a positive probability that a bidder bids the same for both items, even though the bidder values the items differently. This is in accordance with our model where the bidders always bid the same on both items when faced with the discriminatory format.

Our paper extends the existing literature since it presents a different common value, a multiple demand model with private information. First, there are not many multi-unit models that have a pure common value environment and, second, the way we generate the common value and the signals is novel and, third, we model three different (sealed bid) auction formats, in which both the bids (prices) and the quantities are discrete rather than continuous sets. Not all real world auctions are continuous. And, for the two-player environment,
we analytically solve for equilibrium bid strategies, by using the symmetric Bayesian Nash equilibrium concept, which has otherwise been evasive. However, what we will see in a companion paper, where we experimentally test hypotheses excerpted from this model, the way the common value is modeled may be important for the understanding of the same.

The rest of the article is organized as follows. Section 2 presents the model, section 3 describes the equilibrium, section 4 contains the results and section 5 concludes the paper.

2 The model

2.1 Preliminaries

The seller has an inelastic supply of two homogenous items to sell and the seller’s valuation for both items is zero. There are \( n \) bidders in the game. All bidders assign the same value to each item, i.e. bidders have flat demand, and this (common) value is generated by means of an integer generator. More precisely, this is done by making two independent draws from the uniform distribution with support \( D = \{1, 2, \ldots, 6\} \) and adding the two numbers together.\(^3\)

Thus, the distribution of the value is the sum of two uniform distributions, \( V = D_1 + D_2 \). The bidders are not fully informed about this value, however. In particular, each bidder \( i \) receives a private signal of the valuation by observing one of the integer numbers (draws), before the bids are submitted. There is equal probability for bidders to see either outcome of the two draws that constitute the value. Moreover, different bidders may (but must not) see different integer numbers. The signal will be interpreted as the player’s type. Each bidder submits two (discrete) sealed bids, specifying a price, but not a particular unit.\(^4\)

\(^3\) This set could be generalized to \( t \), where \( t \in \mathbb{Z}_+ \), in the below analysis. But since we are making use of this smaller set in the equilibrium analysis, we sacrifice generality for simplicity.

\(^4\) Like in footnote 3, we restrict the set of bids, but it could be generalized to the real line in the below analysis.
2.2 The game

Since players are not certain of the characteristics of the other players, viz. the signals, we have an incomplete information game. Hence, we model the game as a Bayesian game.

Such a game consists of:

- A nonempty, finite set of players \( N = \{1, \ldots, n\} \), where \( n \) is the number of bidders. (The seller has no active part in the game.)
- A nonempty, finite set \( \Omega \) of possible states of nature, each of which is a description of the relevant characteristics of all players. If we denote the outcome of the two random variables as \( d = (d_1, d_2) \in \{1, \ldots, 6\}^2 \) and, for each player \( i \in N \), a specification of the observed integer as \( o_i \in \{1, 2\} \), the set of states consists of:

\[
\Omega = \{\omega = (d, o) : \text{\( d = (d_1, d_2) \in \{1, \ldots, 6\}^2 \)},
\text{\( o = (o_1, \ldots, o_n) \in \{1, 2\}^n \)}\right\}
\]

and for each player \( i \in N \) there exists:

1. An action set \( a_i = (a_{i,1}, a_{i,2}) \in A_i = \mathbb{Z}_n^2 \) consisting of bids, rearranged so that it satisfies \( a_{i,1} \geq a_{i,2} \), for the two items being auctioned.
2. A set of signals/types \( T = D = \{1, \ldots, 6\} \) and a signal function/type function \( \tau_i : \Omega \rightarrow T_i \) assigning a signal to each state of the nature. Thus, if the state of nature is \( \omega = (d, o) \), player \( i \) gets to see the signal \( o_i \in \{1, 2\} \) which has the outcome \( d_{o_i} \in \{1, \ldots, 6\} \). Consequently, the signal function is defined, for each \( \omega \in \Omega \), by \( \tau_i(\omega) = d_{o_i} \), the outcome of the observed integer. Thus, \( \tau_i \) has full range.
3. A probability measure, which is common knowledge and equal for all players, \( \mu : \Omega \rightarrow [0, 1] \) over the states of the nature. Since the integers are generated independently and the outcome observed is independently drawn, all states of nature have the same probability:

\[
\forall \omega = (d, o) \in \Omega : \mu(\omega) = \left(\frac{1}{6}\right)^2 \cdot \left(\frac{1}{2}\right)^n.
\]

4. A utility function, or payoff function \( u_i : A \times \Omega \rightarrow \mathbb{R} \), where \( A = A_1 \times \cdots \times A_n \), assigning an associated payoff to each profile of actions and each state of the nature.

A pure strategy for player \( i \) assigns a bid to each of his or her signals, that is a function \( b_i : T_i \rightarrow A_i \) mapping signals into actions, or, more specifically, \( b_i : \{1, \ldots, 6\} \rightarrow \mathbb{Z}_n^2 \).

The game is now fully specified and we continue with a more careful description.
of the payoff function.

2.3 The payoff

The value for each bidder is the realization of the two random variables drawn from \( D \); hence \( V = d_1 + d_2 \). Moreover, denote by \( c_{-i} \) the two-vector of competing bids facing player \( i \). This is obtained by rearranging the \( 2(N - 1) \) bids \( a_j \) of players \( j \neq i \in N \) in decreasing order and selecting the first two of these. Then, the number of units that player \( i \) wins is just the number of competing bids she defeats. (Ties will be randomly broken. This will be the rule henceforth for all kinds of ties, such as the highest losing bid etc.)

The price paid in the each auction is a function mapping actions and auction forms into a price, i.e. \( p^l : A \to \mathbb{R} \) where \( l \in \{U, D, V\} \); \( U \) stands for the uniform, \( D \) for the discriminatory and \( V \) for the Vickrey pricing rule, respectively.

In the discriminatory auction, each player pays an amount equal to the sum of her bids that are deemed to be winning - that is, the sum of her bids that are among the two highest of the \( N \times 2 \) bids submitted in total. Formally, if exactly \( k_i \) of player \( i \)'s two bids, \( b_{i,1} \) and \( b_{i,2} \), are among the two highest of all bids received, then player \( i \) pays

\[
p^D_i(b) = \sum_{j=1}^{k_i} b_{i,j}.
\]

In all three payment rules, we adopt the convention that if \( k_i \) is equal to zero, the sum will be zero.

Both objects are sold at the market clearing price in the uniform auction. This price is defined to be equal to the highest losing bid, that is

\[
p^U_i(b) = \max\{b_{i,2}, c_{-i,3} - k_i\} k_i
\]

where \( c_{-i,3} \) is set to zero. (This happens when \( k_i = 0 \), but then the player gets zero units and is therefore not supposed to pay anything anyway.)

In the Vickrey auction, a player who wins \( k_i \) units pays the \( k_i \) highest losing bids of the other players - that is, the \( k_i \) highest bids not including her own. Hence, the winner is asked to pay an amount equal to the externality she exerts on other competing bidders. Thus, if player \( i \) wins \( k_i \) units, then the amount she pays is
\[ p_i^V(b) = \sum_{j=1}^{k_i} c_{i,2-k_i+j}. \] (3)

A crucial variable in the payment rule above and in the utility function below is the number of item(s) won, which is a function of the bids \( k_i = k_i(b) \). It takes the value of zero, one or two depending on whether the player wins zero, one or two items, respectively.

Thus, the payoff is the number of items won multiplied by the value of these items, minus the price the winner must pay for them.

\[ u_i(b, \omega) = k_i(b)[d_1 + d_2] - p_i^V(b). \] (4)

If a bidder achieves zero items, then \( k_i \) is zero and hence, \( p_i^V \) becomes zero; thus she also obtains zero payoff.

### 2.4 Example: Payoff in the uniform price auction

For simplicity, we will adopt the rule that if a player bids zero on one or both of her bids, the interpretation will be that she does not want those items. Thus, if all bid zero, the seller will not sell the items. But a zero payment is not ruled out.\(^5\)

Denote by \( B(b) \in \mathbb{R}_+^2 \) the vector of all bids in decreasing order, where \( B_1(b) \) is the highest bid and \( B_{2n}(b) \) the lowest. According to equation 2 above, we have the following payment rule for the uniform auction: \( p_i^U = \max\{b_{i,2}, c_{i,3-k_i}\} k_i \).

There are five different cases that can arise:

1. \( B_1(b) = 0 \). Everybody bids zero. Then, since \( b_i = (0,0) \) for all \( i \in N \), no one gets (wants) the items, i.e. \( k_i = 0 \) for all \( i \in N \), and, therefore, he/she pays nothing. Hence, \( u_i(b, \omega) = 0 \) for all \( i \in N \).
2. \( B_1(b) > 0 \), \( B_2(b) = 0 \). Then, only one item is sold, and its price, the highest losing bid, is zero. The bidder, say \( j \), who wins the object has

\(^5\) That is, the bids will count when finding the price for the units, but not as an active bid. For example, in the Vickrey auction, where the winner always pays a price that comes from a bid that someone else has placed, it could be a zero payment if the price setting bid is zero. A price setting bid equal to zero is also possible in the uniform auction, but not in the discriminatory auction, since the winner pays her own bid in this auction, and a zero bid is defined as not wanting the unit.
$k_j = 1$, the rest, $i \neq j \in N$ have $k_i = 0$. Hence, for all $i \in N$

$$u_i(b, \omega) = \begin{cases} 
0 & \text{if } b_{i,1} = 0, \\
\frac{d_1 + d_2}{2} & \text{if } b_{i,1} > 0.
\end{cases}$$

(3) $B_1(b) \geq B_2(b) > B_3(b) \geq 0$. Now both items are sold at the price $B_3(b)$. Hence, for all $i \in N$

$$u_i(b, \omega) = \begin{cases} 
0 & \text{if } b_{i,1} < c_{-i,2}, \\
(d_1 + d_2) - B_3(b) & \text{if } b_{i,1} > c_{-i,2} \text{ and } b_{i,2} < c_{-i,1}, \\
2(d_1 + d_2) - 2B_3(b) & \text{if } b_{i,2} > c_{-i,1}.
\end{cases}$$

Note that $B_3(b) = \max_{j \in N} \{b_{j,k_j+1}\}$, the same operator as in the pricing rule.

(4) $B_1(b) > B_2(b) = B_3(b) > 0$. Here, we have a clear winner for one of the items, but there is a tie for the second item. Let $m$ be the number of bids that is equal to $B_2(b)$. Then, for all $i \in N$

$$u_i(b, \omega) = \begin{cases} 
0 & \text{if } b_{i,1} < c_{-i,2}, \\
\frac{d_1 + d_2}{m}((d_1 + d_2) - B_3(b)) & \text{if } c_{-i,1} > b_{i,1} \geq c_{-i,2} > b_{i,2}, \\
\frac{d_1 + d_2}{m}((d_1 + d_2) - B_3(b)) & \text{if } c_{-i,1} > b_{i,1} = b_{i,2} \geq c_{-i,2}, \\
\frac{m + 1}{m}((d_1 + d_2) - B_3(b)) & \text{if } b_{i,1} > b_{i,2} \geq c_{-i,1} \geq c_{-i,2}.
\end{cases}$$

(5) $B_1(b) = B_2(b) = B_3(b) > 0$. Once more, there are multiple potential winners for the items. Hence, if $m$ denotes the number of those, we have, for all $i \in N$

$$u_i(b, \omega) = \begin{cases} 
0 & \text{if } b_{i,1} < c_{-i,2}, \\
\frac{d_1 + d_2}{m}((d_1 + d_2) - B_3(b)) & \text{if } b_{i,1} \geq c_{-i,1} \geq c_{-i,2} > b_{i,2}, \\
\frac{d_1 + d_2}{m}((d_1 + d_2) - 2B_3(b)) & \text{if } b_{i,1} = b_{i,2} \geq c_{-i,1} \geq c_{-i,2}.
\end{cases}$$

In this example, where $B_3(b)$ is the price to pay for each item, one can see the rationale for demand reduction which means that a bidder has incentives to shade the bid for the second unit.\footnote{The issue of demand reduction has been studied in more detail by Ausubel and Crampton (2002).} In a uniform price auction, the shading of bids for units other than the first results from the fact that, with positive probability, the bid on the second unit may determine the price paid for the first unit (if it becomes the market-clearing price).
In other words, a bidder’s own bid influences the price paid for all units. The demand reduction property may also result in a bidder not winning (or bidding on) her second object, even though she could have made a positive profit on that object. This complicates the analysis and can, as we will see, give rise to some extreme equilibrium strategies.

3 Equilibrium

A strategy profile \( b = (b_1, \ldots, b_n) \) is a Bayesian (-Nash) equilibrium if, for each player \( i \in N \) and for each of her types \( t_i \in T_i \), the bid \( b_i(t_i) \) is optimal in the sense that it maximizes her expected payoff, given the bidding functions of her opponents and her updated beliefs conditional on her signal.

Let the vector of strategies of all players except player \( i \) be denoted by \( b_{-i} = \{b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n\} \). Let also the vector of all players’ type except player \( i \)’s type be denoted by \( t_{-i} = (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n) \). Then, since all types have positive probabilities, due to the full range of the type function \( \tau_i \), the bid/strategy \( b_i \) is a (pure strategy) Bayesian equilibrium if player \( i \) maximizes her expected utility conditional on \( t_i \) for each \( t_i \):

\[
\pi^i(b_i(t_i)) = \sum_{t_i=1}^{6} \cdots \sum_{t_{i-1}=1}^{6} \sum_{t_{i+1}=1}^{6} \cdots \sum_{t_n=1}^{6} \mu_i(t_{-i}|t_i) E[u_i(b, v)].
\]

where \( \mu_i(t_{-i}|t_i) \) is the posterior belief, see below, about the other types conditional on the player’s own type, \( E \) is the expectation operator, and the summation is over each of player \( i \)’s opponents. \(^7\)

3.1 Posterior beliefs

Since for all \( i \), \( \{\tau_i^{-1}(t_i)\}_{t_i \in T_i} \) is a partition of \( \Omega \), we can identify each \( t_i \in T_i \) with the cell of the partition of \( \Omega \) where the signal \( t_i \) is received. The interpretation we follow is that once a player \( i \) receives the signal \( t_i \in T_i \), she deduces that the state is in the set \( \tau_i^{-1}(t_i) = \{\omega \in \Omega | \tau_i(\omega) = t_i\} \). Her posterior belief about the state that has been realized can then be updated by Bayes’ rule; that is, she assigns to each state \( \omega \in \tau_i^{-1}(t_i) \) the probability of \( \omega \) conditional on

\(^7\) There is also a restriction on the summations due to the fact that only two integers are used. That is, all players together can never have more than two different values for their signals.
\( \tau_i^{-1}(t_i) \), thus:

\[
\mu_i(\omega|t_i) = \frac{\mu(\omega)}{\mu(\tau_i^{-1}(t_i))},
\]

This is identical across players, only the types matter.

Thus, upon observing her own type, a player can also update her beliefs about the opponents’ types by Bayes’ rule. When \( i \) observes \( t_i \), the application of Bayes’ rule yields the conditional probability of the opponents’ types, given \( i \)’s type, \( i \)’s posterior belief \( \mu_i \), by

\[
\mu_i(t_{-i}|t_i) = \frac{\sum_{\omega \in \tau_i^{-1}(t_i)} \mu(\omega)}{\sum_{\omega \in t_i} \mu(\omega)} = \begin{cases} 
\frac{1}{2^{n-1}} \cdot 1 + \frac{2^n-1}{2^{n-1}} \cdot \frac{1}{6} & \text{all types in } t_{-i} \text{ are equal to } t_i, \\
\frac{(n-1)}{x} \cdot \frac{1}{2^{n-1}} \cdot \frac{1}{6} & \text{At least one type in } t_{-1} \text{ is different from } t_i, \\
\frac{1}{2^{n-1}} \cdot \frac{1}{6} & \text{all types in } t_{-i} \text{ are different from } t_i.
\end{cases}
\]

where \( x \in \{0, \ldots, n-1\} \) is defined as the number of players of the same type as player \( i \). The first equation in the second row works as follows: If all other players, except player \( i \), see the same value as \( t_i \), two things can occur. Either they all see the same integer as player \( i \), which happens with probability \( \frac{1}{2^{n-1}} \), or at least one of the others sees a different integer with the same value as \( t_i \), which happens with probability \( \frac{2^n-1}{2^{n-1}} \cdot \frac{1}{6} \). (The first term is the probability that at least one player sees a different integer and the second term is the probability of that integer having the same value as \( t_i \).

The second equation says that if one of the non-\( i \) players sees a different value than player \( i \), the belief for player \( i \) becomes \( \left(\frac{n-1}{x}\right) \cdot \frac{1}{2^{n-1}} \cdot \frac{1}{6} \). In the last row, all non-\( i \) players see a different value than player \( i \), and, since we only have two integers, they all see the same integer. This occurs with probability \( \frac{1}{2^{n-1}} \cdot \frac{1}{6} \).

### 3.2 Conditional expected value functions

Since players only get to see one integer, i.e. their signal, they will have to use the expected value when calculating their value, which is \( v(t_i) = t_i + \frac{1}{6} \sum_{j=1}^{n} t_j = t_i + \frac{7}{6} \), that is, the value of her signal plus the expected value of the other integer. But in a Bayesian game, they will also need to calculate their competitors’ value, given their own signal. This conditional expected value for the other players is dependent on how many players there are in the auction.

The fact that induces this is that they can all see the same integer or different integers. The more signals (players), the more accurate becomes the
conditional expected value. That is, with many bidders, we approach the true value. This is an application of information aggregation, studied by Wilson (1977). The conditional expected value is defined as:

\[
v_i(t_{-i}|t_i) = v(t_{-i}|t_i)
\]

\[
= \begin{cases} 
\frac{1}{2n+1}(t_i + \frac{7}{2}) + \frac{2n-1}{2n+1} \cdot 2t_i & \text{all } t_j = t_i, \\
t_i + t_j & \text{(where } t_j \in t_{-i}) \text{ some } t_j \neq t_i, 
\end{cases} 
\]  
\[ (6) \]

\[
= \begin{cases} 
\frac{2n-1}{2n+1}t_i + \frac{1}{2n+1} \cdot \frac{7}{2} & \text{all } t_j = t_i, \\
t_i + t_j & \text{(where } t_j \in t_{-i}) \text{ some } t_j \neq t_i. 
\end{cases} 
\]  
\[ (7) \]

The first row in equation (6) says that if the non-i players are of the same type, \( t_i \), as player i, two things can happen. Either they see the same integer as player i, which occurs with probability \( \frac{1}{2n+1} \), or at least one of them sees a different integer. The value for the former becomes \( t_i + \frac{7}{2} \) for player i, while the value for the latter becomes \( t_i + t_i = 2t_i \).

In the second row of the same equation, we see the value if one player, or both, is of another type than player i. Then, since there are only two distinct integers, the value becomes the sum of the integer values. Equation (7) is just a simplification.

The above term expectation means the expectation of the possible outcomes of the integer values. From player i’s perspective, if we also take expectations over all \( t_{-i} \), we get the expected value for a competitor given player i’s type. That is, we must combine the posterior beliefs with the conditional expected value to get the expected value for any competitor. Hence, the expected value for a competitor for player i is defined as

\[
E_{t_{-i}}[v(t_{-i}|t_i)] = \sum_{t_{-i}} \mu(t_{-i}|t_i)v(t_{-i}|t_i). 
\]  
\[ (8) \]

If we also take the expectation of all \( t_i \), the terms will sum up to 7 as they should. But for any given \( t_i \), the expected value for a competitor will not be \( t_i + 7/2 \) since we have to take into account that the competitor may get the signal from the same integer as player i.

Equation 5 can then be stated as, for \( l \in \{D, U, V\} \), the bid \( b_i \) is a (pure strategy) Bayesian equilibrium, if player i maximizes her expected utility conditional on \( t_i \) for each \( t_i \):

12
\[ \pi_l(b_i(t_i)) = \sum_{t_1=1}^{6} \cdots \sum_{t_{i-1}=1}^{6} \sum_{t_{i+1}=1}^{6} \cdots \sum_{t_n=1}^{6} \mu(t_{-i}|t_i) [v(t_{-i}|t_i) k_i(b'_i, b_{-i}) - p_l'(b'_i, b_{-i})]. \]

The summation is over each of player \(i\)'s opponents. 8

4 Results

Since the value function is symmetrical and since we have a symmetrical joint distribution, only types will matter when bidding; thus, we will look for a symmetrical equilibrium. We start with a two-player game.

4.1 Two players

For the case with two players, the objective function (eq. 9) to optimize for player \(i \in N\), for each \(t_i\), and for auction formats \(l \in \{D, U, V\}\), is

\[ \pi_l(b_i(t_i)) = \sum_{t_j=1, j \neq i}^{6} \frac{1}{12} [(t_i + t_j) k_i(b_i(t_i), b_j(t_j)) - p_i(b_i(t_i), b_j(t_j))] + \frac{7}{12} \left[ (\frac{3}{2} t_i + \frac{7}{4}) k_i(b_i(t_i), b_j(t_i)) - p_i(b_i(t_i), b_j(t_i)) \right] \]

where \(b_j(t_i) = b_i(t_i)\) for all \(t_i \in T_i\) by the symmetry conjecture. That is, given equal signals, different players choose the same strategy.

Since the pricing rule is different for the three auction formats, which implies different strategies for each auction, we have to separate the analyzes of the the three formats. We will start with the discriminatory auction.

4.1.1 The discriminatory auction

In this auction, conditional on winning, for each item won, every bidder pays the price of her bid on that item (cf. equation (1) on page 7).

**Proposition 1** The unique pure strategy Bayesian equilibrium profile of this game is

8 With the restriction that all players together can never have more than two different values for their signals.
\[ b^*(t_i) = (t_i + \left[ \frac{t_i}{3} \right], t_i + \left[ \frac{t_i}{3} \right]), \]

where \( \left[ x \right] \) is a ceiling function which maps \( x \) to the smallest following integer, i.e. \( \left[ x \right] = \min\{n \in \mathbb{Z} | n \geq x\} \).

**Proof 1** See proof 8 in the Appendix on page 23

If we then assume that there is equal probability of getting either signal, from equation 16 (in section 7.1.1), we have that a bidder’s expected payoff for a game with two players is:

\[
E_{\pi_i}^D(b^*) = \frac{1}{6} \sum_{i=1}^{6} \pi_i = \frac{25}{18} \approx 1.39.
\]

**4.1.2 The Uniform price auction**

The uniform auction differs from the discriminatory auction described above by the pricing rule, which is defined as the highest losing bid of all bidders (cf. equation (2) on page 7). This dissimilarity makes a major difference in strategies, and, hence, equilibria.

We now have a multitude of equilibria, but they have one thing in common; namely, the bid for the first unit is always the same for each type of player. Still, this comes as no surprise since bidders have a weak incentive to bid their full value on the first unit in independent private value auctions, cf. Nousair (1995). This is not entirely true in this setting due to the constraint of integer bids and the fact that the value of the objects is the same for all players. Here, the first unit bid is the smallest following integer of the conditional expected value, given the bidder’s signal. Hence, from equation 8, we have:

\[ b^*_1(t_i) = \left\lceil E_{t_{-i}}[v(t_{-i}|t_i)] \right\rceil. \]

However, bidders have incentives to shade their bids for additional units since, with positive probability, the bid on the second unit may determine the price paid on all units.

The worst that can happen for the seller is what is known as extreme demand reduction, where all bidders act as a single-unit demander and bid zero on the second unit, which gives the seller zero revenue. In this setting, the zero bid is indeed an equilibrium and, since it also gives the most payoff to the players, it may also count as the most preferable equilibrium for them. Hence, we argue that no other equilibrium payoff dominates.
Proposition 2

\[ b^*(t_i) = (b^*_1, b^*_2) = ([E_{t_i}[v(t_{-i}|t_i)]], 0). \]

**Proof 2** Suppose that player \( j \) utilizes \( b^*(t_i) \). Any attempt to win 2 units for player \( i \) would either make her second unit bid, or the first unit bid of player \( j \), set the price, and since the first unit bid from player \( j \) is \( b^*_1(t_j) \geq [E_{t_j}[v(t_{-j}|t_j)]] = 4 \), player \( i \) must bid at least 5 to win. The payoff for using \( b^*(t_i) \) is the expected value minus the price paid, which is zero, hence \( \pi^* = t_i + 7/2 \), while the expected value for using the alternative strategy would be \( \pi' \leq 2(t_i + 7/2 - 5) \). Then, we have that \( \pi' > \pi^* \) implies (at best) \( 2(t_i + 7/2 - 5) > (t_i + 7/2) \Rightarrow t_i > 6 \), which is impossible.

Thus, from the seller’s point of view, uniform price auctions may have undesirable properties.

But other equilibria also exist. First, any bid on the first unit above the conditional expected value is an equilibrium bid\(^9\). Second, if both bidders bid 1, 2 or 3 on the second unit, irrespective of \( t_i \), it is also an equilibrium bid. But, since it is highly unclear on which of these equilibria the bidders would coordinate, the zero-bid on the second unit is clearly focal on behalf of its payoff-determining equilibrium in undominated strategies.

Using the focal equilibrium strategy, we have that a bidder’s expected payoff for a game with two players is

\[ E\pi^U_i = \frac{1}{6} \sum_{i=1}^{6} \pi_i = 7, \]

the expected value of the two integers.

If the players instead played the best the seller could hope for, i.e. 3, on the second unit, a bidder’s expected payoff would be

\[ E\pi^U_i = \frac{1}{6} \sum_{i=1}^{6} \pi_i = 4. \]

4.1.3 The Vickrey auction

In the Vickrey auction, the number of units won for a bidder is equal to the number of competing bids that she defeats. Likewise, the prices that she pays are determined by the competing bids she defeats.

\(^9\) Levin (2005) has showed that, in an IPV setting with a reservation price equal to zero, any bid on the first unit weakly above the endpoints of the value-distribution is an equilibrium bid, as long as there are as many bidders as units for sale.
The Vickrey auction has (no more than) two equilibria, which are the two extreme strategies, on each side of the market. First there is the equilibrium most preferable to the seller, the demand-revealing equilibrium and, on the other side, the most preferable equilibrium for the buyer, the extreme demand-reduction equilibrium.

**Proposition 3** The demand-revealing equilibrium is

\[
 b^*(t_i) = (b^*_1, b^*_2) = (t_i + \lceil \frac{t_i}{2} \rceil + 2, t_i + \lceil \frac{t_i}{2} \rceil + 1),
\]

where \(\lceil \cdot \rceil\) is defined as above.

The reason for different bids on the two units is purely technical and has its origins in the restriction to only bid in integers. The conditional expected value is (for all signals) between bid 1 and bid 2, i.e. \(b^*_1 > v_i(t_{-i}|t_i) > b^*_2\). Thus, it has nothing to do with demand reduction.

**Proof 3** Suppose that bidder \(i\) wins \(k_i\) units when both bidders follow the proposed strategy. That is, exactly \(k_i\) of her bids are among the two highest bids of both players. If the type-\(t_i\) bidder bids less on one or both units, then the number of units that she wins is at most what she would win by bidding \(b^*(t_i)\). For any of the units won, the prices will be the same as before, but she would forgo some expected surplus for units that she did not win.

If she instead bids higher on one or both of her units, then she wins at least as many units as before. The prices for the first \(k_i\) units will remain the same as if she bids \(b^*(t_i)\). For any additional units, however, the price paid will be too high, since for \(k > k_i\) the price is greater than the expected value of the item(s); which is seen by \(b^*_1 > v_i(t_{-i}|t_i) > b^*_2\).

**Proposition 4** The extreme demand-reduction equilibrium is

\[
 b^*(t_i) = ([E_{L_i}[v(t_{-i}|t_i)]], 0).
\]

**Proof 4** The proof is as in the uniform auction, hence it is omitted.

There is a much weaker equilibrium strategy in the Vickrey auction than in the uniform auction because, if player \(i\) bids the above strategy in the uniform auction, player \(j\)’s best response is to bid the same. That is not entirely true in the Vickrey auction, since you never pay what you bid, but what the other player bids. Hence, in the Vickrey auction, player \(j\) can bid any number below her conditional expected value for the second unit and still be an equilibrium strategy. And, by the same token, any bid below the conditional expected value is an equilibrium bid.
For this equilibrium, as in the uniform auction, we have that any bid on the first unit above the conditional expected value is an equilibrium bid.

Consequently, using the payoff-dominating equilibrium strategy, the expected payoff for a game with two players is

\[ E^{\pi_i}_V = \frac{1}{6} \sum_{i=1}^{6} \pi_i = 7, \]

the same as for the uniform auction, whereas, using the demand-revealing equilibrium strategy gives

\[ E^{\pi_i}_V = \frac{1}{6} \sum_{i=1}^{6} \pi_i = \frac{85}{72} \approx 1.18. \]

4.1.4 Revenues

In a (pure) common value auction, the revenue is strongly negatively correlated with the profit. Above, we saw that both the uniform and the Vickrey auctions had an equilibrium that gave all surplus to the buyer, which was the same as the expected value of the two integers, i.e. 7. This translates into zero revenue to the seller. The discriminatory auction, on the other hand, had a unique equilibrium that gave the expected profit of 1.39 to the buyers.

To find the expected revenue in the discriminatory auction, we calculate the probability for each set of possible joint signals between the players. We make use of the strategies implicitly inherent in the signals to compute the price paid for each possible set of joint signals. Then, we have the expected revenue as the product of the intersection of the signals and the realized price in that outcome. For two players, player \( i \) and player \( j \), it becomes:

\[ E[R] = P(t_i \cap t_j)p(b_i, b_j), \quad (13) \]

where \( p(b_i, b_j) \) is the price paid. If there are three players, we instead calculate \( P(t_i \cap t_j \cap t_k)p(b_i, b_j, b_k) \), and so on. Then, we get that the expected revenue is 2.09 in the discriminatory auction.

Consequently, the discriminatory auction delivers the greatest revenue to the seller. It is a close game between the uniform and the Vickrey auction. Nonetheless, because of the existence of the Vickrey auction’s demand-revealing equilibrium, the Vickrey auction may be ranked higher than the uniform auction.
4.2 Three or more players

In the discriminatory auction, when we increase the bidders by one, all bidders but the type-6 player bid the same as in a two-player game (see section 7.1.2). The type-6 players raise their bids on both units by one increment unit due to more fierce competition.

\[
u_i(b, \omega) = \begin{cases} 
(t_i + \lceil \frac{t_i}{3} \rceil, t_i + \lfloor \frac{t_i}{3} \rfloor) & t_i \in \{1, 2, 3, 4, 5\}, \\
(9, 9) & t_i = 6.
\end{cases}
\]

The expected payoff decreases for all players which, in turn, raises the revenue for the seller.

In a four-player game, we get the same increase in the bids of the type-6 players as in the three-player game (see section 7.1.3), but a decrease in the bids of the type-1 players. The decrease is one increment unit. The optimal strategy can then be written as:

\[
b^*(t_i) = (t_i + \lceil \frac{t_i}{2} \rceil, t_i + \lfloor \frac{t_i}{2} \rfloor),
\]

where \(\lfloor x \rfloor\) is a floor function which maps \(x\) to the largest previous integer, i.e. \(\lfloor x \rfloor = \max\{m \in \mathbb{Z} | m \leq x\}\).

Thus, when the number of players increases, the bids can either increase or decrease but the expected revenue always increases with the number of participants in the auction. For a game with three players, the expected revenue is 2.69, and for a game with four players, it is 2.82 (in the two-player game, it was 2.11).

Thus, we see two counteracting effects on equilibrium bidding behavior. The increasing bids reflect the competitive effect; the more bidders, the greater the competition. The decreasing bids reveal, on the other hand, the winner’s curse effect; the more bidders, the greater the potential winner’s curse. It is also readily visible that the high-value-bidders belong to the former category and the low-value-bidders to the latter category. And for the seller it is better to have more bidders than a few, as the competitive effect dominates the winner’s curse effect.

For more bidders, the calculations become more complicated, but one can see from the model that increasing the number of bidders even more would induce the low-value-bidders to drop out of the auction, due to a negative expected payoff. High-value-bidders would, of course, anticipate this and bid according
to that, but it is hard to say in what way bids will change without doing a numerical calculation.

When there are more than two players in the uniform auction, two things occur. First, we have to correct downwards instead of upwards, because now there is a chance that someone’s first-unit bid may become the price setting bid. And, second, as a result of the first, the zero bid on the second unit is no longer an equilibrium. This is due to the fact there are now at least three bidders and two units, and all three bidders have a weak incentive to bid the true (expected) value of the first unit.

**Conjecture 5 (More than two bidders)** When there are more than two bidders in the auction, the pure Bayesian equilibrium strategy is to bid the following on the first unit:

\[ b^*_1(t_i) = \lceil E_{t_{-i}}[v(t_{-i}|t_i)] \rceil. \] (15)

where \( \lceil x \rceil \) is defined as above.

**Proof 5** First, note that to bid more than \( b^*_1 \) will incur an expected loss if the bid is above both \( E_{t_{-i}}[v(t_{-i}|t_i)] \) and the price. That is, suppose that player \( i \) bids \( b'_1 > E_{t_{-i}}[v(t_{-i}|t_i)] \). Then, if \( b'_1 > p > E_{t_{-i}}[v(t_{-i}|t_i)] \), a loss of \( p - E_{t_{-i}}[v(t_{-i}|t_i)] \) will be realized on that unit.

Second, suppose that the bid is below the equilibrium bid \( b'_1 < b^*_1 \). Then, three cases appear, first if the bid is below the value which, in turn, is weakly below the price, i.e. \( p \geq E_{t_{-i}}[v(t_{-i}|t_i)] > b'_1 \). Then, nothing would change if the player were to raise the bid to \( E_{t_{-i}}[v(t_{-i}|t_i)] \). Next, if the bid is below \( E_{t_{-i}}[v(t_{-i}|t_i)] \) and above the price, \( E_{t_{-i}}[v(t_{-i}|t_i)] > b'_1 > p \). Nothing would change here either if the bid were increased to \( E_{t_{-i}}[v(t_{-i}|t_i)] \). The last case is if the value is greater than the price and the price is weakly greater than the bid, \( E_{t_{-i}}[v(t_{-i}|t_i)] > p \geq b'_1 \). Now, if the player raised the bid to \( E_{t_{-i}}[v(t_{-i}|t_i)] \), she would win a unit at a profitable price. Thus, to bid the proposed equilibrium bid on the first unit is (weakly) dominant in expectation.

Now, by the last conjecture, when there are more than two players, the bid on the second unit will be weakly bounded from below by the first-unit bid from the low type player. Thus

**Conjecture 6 (More than two players)** The second unit bid is weakly bounded by 4, i.e. \( b_2(t_i) \geq b^*_1(1) = E_{t_{-i}}[v(t_{-i}|1)] = 4 \).

**Proof 6** If player \( i \), say, bids below 4, she will win at most one unit and get the payoff: \( \pi'_i = (t_i + 7/2 - p)k_i' \), where \( k_i' \leq 1 \). While, if the player bids 4, the payoff will be: \( \pi'_i = (t_i + 7/2 - p)k_i^* \), where \( k_i^* \geq k_i' \) since the bid \( b^*_1(t_i) \) now competes against the other bids which the zero-bid (in 2-player games)
did not. And, since the bid does not affect the price, \( p \) will be the same in both payoff functions above. Hence, \( \pi_i \geq \pi_i' \).

**Conjecture 7 (Many players)** The more bidders in the auction, the higher the bids. This is true for both the first-unit bid and the second-unit bid.

**Proof 7** Given any realization of the two dice, we see from equation (7) that the conditional expected value weakly increases with the number of players. And, as can be seen from conjectures 5 and 6, since both the first-unit and the second-unit bids are dependent on that value, we have that both bids increase with the number of players.

In the Vickrey auction, when there are more than two bidders, all pure equilibria disappear. This is an example of a non-core outcome; that is, there is always a coalition that can get a greater payoff by blocking all proposed outcomes. This complication emanates from the discrete setting of the formats, or, to be more precise, due to the restriction to only bid in integer values.

## 5 Conclusion

Analytically identifying equilibrium strategies in a common value auction when more than one unit is demanded is hard due to its complex nature. Here, we build a model where strategies can be found and analyzed, at least for a few bidders. We run into the problem of a non-core outcome in the Vickrey auction when we have more than two players.

For two-player games, the discriminatory auction delivers the greatest expected revenue. Due to their zero-revenue equilibria, both the uniform and the Vickrey auctions are far from the discriminatory auction in revenue ranking. The Vickrey auction is a little better at surrendering revenue as compared to the uniform auction, since its zero-revenue equilibrium is weaker and because of its second, demand-revealing, equilibrium. But the Vickrey auction is rarely implemented in real life auctions because of its complicated nature; see, for example, Ausubel and Milgrom (2006).

For the discriminatory auction, we are able to find strategies for a larger number of bidders. We see, for example, that both high signal carriers and low signal carriers change their strategies when we increase the number of bidders in the auction; bidders with a low signal reduce their bids and high signal bidders raise theirs. Since the probability of winning is low for the low signal holders, we see an increase in revenue when increasing the number of bidders.
In both the uniform and the Vickrey auctions, we see that bidders have strong incentives for demand reduction; if they bid the indicated value for the first unit and zero for the second, the seller gets zero revenue. Hence, as long as there are only two bidders, these can secure one unit each at the price of zero. In a small-scale experiment, conducted in 2007 with master’s students at Örebro University, we encountered this behavior in some sessions. A subject for further research is to test the model in a more thorough laboratory setting and see whether the winner’s curse conjecture stated in the introduction is valid.

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References


7.1 Objection functions, equilibria and payoffs in the discriminatory auction

We only allow integer-value-bids, \((b_1, b_1, b_2) \in \mathbb{Z}^2_+\), and let \(t = 6\). Since the value function is symmetrical and we have a symmetrical joint distribution, only types will be of importance when bidding; thus, we will look for a symmetrical equilibrium.

7.1.1 2 players

\[
π_l(b_i(t_i)) = 7 \frac{12}{4} [3t_i + 7 \frac{3}{4}] k_i(b_i(t_i), b_j(t_i)) − p_i(b_i(t_i), b_j(t_i)) + \sum_{j=1, j \neq t_i} 12 [t_i + j] k_i(b_i(t_i), b_j(j)) - p_i(b_i(t_i), b_j(j)).
\]

where \(b_j(t_i) = b_i(t_i)\) for all \(t_i \in T_i\).

The discriminatory auction’s pure Bayesian equilibrium strategy profile with adherent payoff from this game is:

1. \(b^*(1) = (2, 2) \Rightarrow π_1(2, 2) = 35 \frac{48}{48} \approx 0.73\)
2. \(b^*(2) = (3, 3) \Rightarrow π_2(3, 3) = 49 \frac{48}{48} \approx 1.02\)
3. \(b^*(3) = (4, 4) \Rightarrow π_3(4, 4) = 71 \frac{48}{48} \approx 1.48 (16)\)
4. \(b^*(4) = (6, 6) \Rightarrow π_4(6, 6) = 49 \frac{48}{48} \approx 1.02\)
5. \(b^*(5) = (7, 7) \Rightarrow π_5(7, 7) = 79 \frac{48}{48} \approx 1.65\)
6. \(b^*(6) = (8, 8) \Rightarrow π_6(8, 8) = 39 \frac{16}{16} \approx 2.44\)

Proof

The equilibrium prescribes equal bids on both units and the payoff is:
7 Appendix

7.1 Objection functions, equilibria and payoffs in the discriminatory auction

We only allow integer-value-bids, \((b_{i,1}, b_{i,2}) \in \mathbb{Z}_+^2\), and let \(t = 6\). Since the value function is symmetrical and we have a symmetrical joint distribution, only types will be of importance when bidding; thus, we will look for a symmetrical equilibrium.

7.1.1 2 players

\[
\pi^i(b_i(t_i)) = \frac{7}{12} \left( \frac{3}{2} t_i + \frac{7}{4} \right) k_i(b_i(t_i), b_j(t_j)) - p_i(b_i(t_i), b_j(t_j)) \right] \\
\quad + \sum_{j=1, j \neq i}^{t} \frac{1}{12} [(t_i + j)k_i(b_i(t_i), b_j(t_j)) - p_i(b_i(t_i), b_j(t_j))].
\]

where \(b_j(t_i) = b_i(t_i)\) for all \(t_i \in T_i\).

The discriminatory auction’s pure Bayesian equilibrium strategy profile with adherent payoff from this game is:

\[
\begin{align*}
    b^*(1) &= (2, 2) \quad \Rightarrow \quad \pi_1(2, 2) = \frac{35}{48} \approx 0.73 \\
    b^*(2) &= (3, 3) \quad \Rightarrow \quad \pi_2(3, 3) = \frac{49}{48} \approx 1.02 \\
    b^*(3) &= (4, 4) \quad \Rightarrow \quad \pi_3(4, 4) = \frac{71}{48} \approx 1.48 \\
    b^*(4) &= (6, 6) \quad \Rightarrow \quad \pi_4(6, 6) = \frac{49}{48} \approx 1.02 \\
    b^*(5) &= (7, 7) \quad \Rightarrow \quad \pi_5(7, 7) = \frac{79}{48} \approx 1.65 \\
    b^*(6) &= (8, 8) \quad \Rightarrow \quad \pi_6(8, 8) = \frac{39}{16} \approx 2.44.
\end{align*}
\]

**Proof 8 (Proof)** The equilibrium prescribes equal bids on both units and the payoff is:
\[ \pi(b^*) = \frac{1}{12} \sum_{j=1}^{t_i-1} 2 \left[ (t_i + j) - \left( t_i + \left\lfloor \frac{t_i}{3} \right\rfloor \right) \right] + \frac{7}{12} \left[ \left( \frac{3}{2} t_i + \frac{7}{4} \right) - \left( t_i + \left\lfloor \frac{t_i}{3} \right\rfloor \right) \right] \]

\[ = \frac{1}{12} \sum_{j=1}^{t_i-1} 2 \left[ j - \left\lfloor \frac{t_i}{3} \right\rfloor \right] + \frac{1}{12} \left[ \frac{49}{4} + \frac{7}{2} t_i - 7 \left\lfloor \frac{t_i}{3} \right\rfloor \right] \]

\[ = \frac{1}{12} \sum_{j=1}^{t_i-2} 2 \left[ j - \left\lfloor \frac{t_i}{3} \right\rfloor \right] + \frac{1}{12} \left[ \frac{41}{4} + \frac{11}{2} t_i - 9 \left\lfloor \frac{t_i}{3} \right\rfloor \right] \]

We will make use of the last equation when we scrutinize what happens if the bidder deviates by bidding lower, which we do first, while we make use of the middle equation if she deviates by bidding higher.

Suppose that the bidder deviates and bids \( b'(t_i) = b(t_i - l) \), where \( l \in \mathbb{Z}_+ \), i.e., she mimics a type-\((t_i - l)\) player. Then, she will always lose both units when playing against another type-\(t_i\) bidder and, depending on which type she mimics, both will win one unit each when playing against each other. This means that the best she can do is to mimic a type-\((t_i - 1)\) player and thereby get the following payoff:

\[ \pi(b') = \frac{1}{12} \sum_{j=1}^{t_i-2} 2 \left[ (t_i + j) - \left( t_i - 1 + \left\lfloor \frac{t_i}{3} - 1 \right\rfloor \right) \right] \\
+ \frac{1}{12} \left[ (t_i + t_i - 1) - \left( t_i - 1 + \left\lfloor \frac{t_i}{3} - 1 \right\rfloor \right) \right] \\
= \frac{1}{12} \sum_{j=1}^{t_i-2} 2 \left[ j + 1 - \left\lfloor \frac{t_i}{3} - 1 \right\rfloor \right] + \frac{1}{12} \left[ t_i - \left\lfloor \frac{t_i}{3} - 1 \right\rfloor \right] \]

using the relation \( \left\lfloor \frac{t_i}{3} \right\rfloor - 1 \leq \left\lfloor \frac{t_i-1}{3} \right\rfloor \), we get

\[ \pi(b') \leq \frac{1}{12} \sum_{j=1}^{t_i-2} 2 \left[ j + 2 - \left\lfloor \frac{t_i}{3} \right\rfloor \right] + \frac{1}{12} (t_i - 2) + \frac{1}{12} \left[ t_i + 1 - \left\lfloor \frac{t_i}{3} \right\rfloor \right] \\
= \frac{1}{12} \sum_{j=1}^{t_i-2} 2 \left[ j - \left\lfloor \frac{t_i}{3} \right\rfloor \right] + \frac{1}{12} \left[ 4t_i - 8 + t_i + 1 - \left\lfloor \frac{t_i}{3} \right\rfloor \right] \\
= \frac{1}{12} \sum_{j=1}^{t_i-2} 2 \left[ j - \left\lfloor \frac{t_i}{3} \right\rfloor \right] + \frac{1}{12} \left[ 5t_i - 7 - \left\lfloor \frac{t_i}{3} \right\rfloor \right]. \]

Taking the difference between the payoff when a player places the equilibrium bid, and the payoff if she deviates downwards, we have
\[
\pi(b^*) - \pi(b') \geq \frac{1}{12} \left( \frac{41}{4} + \frac{11}{2} t - 9 \left[ \frac{t}{3} \right] - 5t_i + 7 + \left[ \frac{t_i}{3} \right] \right)
\]
\[
= \frac{1}{12} \left( \frac{69}{4} + \frac{1}{2} t_i - 8 \left[ \frac{t_i}{3} \right] \right) > 0, \quad \forall t_i \in T_i.
\]

Since the difference is positive, the bidder will not make any profit by bidding lower than \(b^*\).

On the other hand, the player may deviate by raising the bid as compared to the equilibrium bid; thus, \(b'(t) = b(t_i + l)\), where \(l \in \mathbb{Z}_+\), and thereby mimic a type-(\(t_i + l\)) player. Then, she will always win both units when playing against another type-\(t\) bidder and, depending on which type she mimics, both will win one unit each when playing against each other. This shows that the best she can do is to mimic a type-(\(t_i + 1\)) player, but it is not good enough since she will then only get the following payoff:

\[
\pi(b') = \frac{1}{12} \sum_{j=1}^{t_i-1} 2 \left[ (t_i + j) - \left( t_i + 1 + \left[ \frac{t_i + 1}{3} \right] \right) \right]
\]
\[
+ \frac{7}{12} \left[ \left( \frac{3}{2} t_i + \frac{7}{4} \right) - \left( t_i + 1 + \left[ \frac{t_i + 1}{3} \right] \right) \right]
\]
\[
+ \frac{1}{12} \left[ (t_i + t_i + 1) - \left( t_i + 1 + \left[ \frac{t_i + 1}{3} \right] \right) \right]
\]
\[
= \frac{1}{12} \sum_{j=1}^{t_i-1} 2 \left[ j - 1 - \left[ \frac{t_i + 1}{3} \right] \right] + \frac{14}{12} \left[ \frac{1}{2} t_i + \frac{3}{4} - \left[ \frac{t_i + 1}{3} \right] \right]
\]
\[
+ \frac{1}{12} \left[ t_i - \left[ \frac{t_i + 1}{3} \right] \right]
\]
\[
= \frac{1}{12} \sum_{j=1}^{t_i-1} 2 \left[ j - \left[ \frac{t_i + 1}{3} \right] \right] - \frac{1}{12} 2(t_i - 1)
\]
\[
+ \frac{14}{12} \left[ \frac{1}{2} t_i + \frac{3}{4} - \left[ \frac{t_i + 1}{3} \right] \right] + \frac{1}{12} \left[ t_i - \left[ \frac{t_i + 1}{3} \right] \right]
\]
using the relation \(\left[ \frac{t_i}{3} \right] \leq \left[ \frac{t_i + 1}{3} \right]\), and merging the last three terms, we get

\[
\pi(b') \leq \frac{1}{12} \sum_{j=1}^{t_i-1} 2 \left[ j - \left[ \frac{t_i}{3} \right] \right] + \frac{1}{12} \left[ -2t_i + 2 + 7t_i + \frac{21}{2} - 14 \left[ \frac{t_i}{3} \right] + t_i - \left[ \frac{t_i}{3} \right] \right]
\]
\[
= \frac{1}{12} \sum_{j=1}^{t_i-1} 2 \left[ j - \left[ \frac{t_i}{3} \right] \right] + \frac{1}{12} \left[ \frac{25}{2} + 6t_i - 15 \left[ \frac{t_i}{3} \right] \right]
\]

Then, we have that
Then, if we assume there is equal probability of getting either signal, a bidder’s
payoff is:

\[ \pi(b^*) - \pi(b') \geq \frac{1}{12} \left[ \frac{49}{4} + \frac{7}{2} t_i - 7 \left( \frac{t_i}{3} \right) - \frac{25}{2} - 6t_i + 15 \left( \frac{t_i}{3} \right) \right] \]
\[ = \frac{1}{12} \left[ - \frac{1}{4} - \frac{5}{2} t_i + 8 \left( \frac{t_i}{3} \right) \right] > 0, \quad \forall t_i \in T_i. \]

Thus, no one has an incentive to deviate from the proposed equilibrium strategy.

Then, if we assume there is equal probability of getting either signal, a bidder’s
expected payoff for a game with two players would be:

\[ E_{\pi_i^D} = \frac{1}{6} \sum_{i=1}^{6} \pi_i = \frac{25}{18} \approx 1.39. \]

7.1.2 3 players

\[ \pi(b(t_i)) = \frac{3}{8} \left[ \left( \frac{5}{4} t_i + \frac{7}{8} \right) k_i(b_i(t_i), b_j(t_i), b_l(t_i)) - p_i(b_i(t_i), b_j(t_i), b_l(t_i)) \right] \]
\[ + \sum_{j=1, j \neq i}^{6} \frac{1}{12} \left[ (t_i + j)k_i(b_i(t_i), b_j(t_i), b_l(t_i)) - p_i(b_i(t_i), b_j(t_i), b_l(t_i)) \right] \]
\[ + \sum_{j=1, j \neq i}^{6} \frac{1}{24} \left[ (t_i + j)k_i(b_i(t_i), b_j(t_i), b_l(t_i)) - p_i(b_i(t_i), b_j(t_i), b_l(t_i)) \right], \]

where \( b_j(t_i) = b_i(t_i) \) for all \( t_i \in T_i \).

The discriminatory auction’s unique pure strategy Bayesian equilibrium profile
(for 3 players) with its payoff is:

\[ b^*(1) = (2, 2) \quad \Rightarrow \quad \pi_1(2, 2) = \frac{5}{32} \approx 0.16 \]
\[ b^*(2) = (3, 3) \quad \Rightarrow \quad \pi_2(3, 3) = \frac{11}{32} \approx 0.34 \]
\[ b^*(3) = (4, 4) \quad \Rightarrow \quad \pi_3(4, 4) = \frac{67}{96} \approx 0.70 \]
\[ b^*(4) = (6, 6) \quad \Rightarrow \quad \pi_4(6, 6) = \frac{15}{32} \approx 0.47 \]
\[ b^*(5) = (7, 7) \quad \Rightarrow \quad \pi_5(7, 7) = \frac{95}{96} \approx 0.99 \]
\[ b^*(6) = (9, 9) \quad \Rightarrow \quad \pi_6(9, 9) = \frac{19}{32} \approx 0.59. \]

**Proof 9** It follows the same lines as the one for two players, but it is quite
messy. Therefore, we will omit it here but it can be obtained from the author
upon request.

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And, if we assume equal probability for each type, the expected payoff to a bidder is:

\[ E\pi_i^D = \frac{1}{6} \sum_{i=1}^{6} \pi_i = \frac{13}{24} \approx 0.54. \]

### 7.1.3 4 players

\[
\pi(b(t_i)) = \frac{13}{48} \left[ \left( \frac{15}{8} t_i + \frac{7}{16} \right) k_i(b_i(t_i), b_j(t_i), b_j(t_i), b_k(t_i)) - p_i(b_i(t_i), b_j(t_i), b_j(t_i), b_k(t_i)) \right] + \sum_{j=1, j\neq t_i}^{6} \frac{1}{16} \left[ (t_i + j) k_i(b_i(t_i), b_j(t_i), b_j(t_i), b_j(t_i)) - p_i(b_i(t_i), b_j(t_i), b_j(t_i), b_j(t_i)) \right] + \sum_{j=1, j\neq t_i}^{6} \frac{1}{48} \left[ (t_i + j) k_i(b_i(t_i), b_j(t_i), b_j(t_i), b_j(t_i)) - p_i(b_i(t_i), b_j(t_i), b_j(t_i), b_j(t_i)) \right]
\]

where \( b_j(t_i) = b_i(t_i) \) for all \( t_i \in T_i \).

The discriminatory auction’s unique pure strategy Bayesian equilibrium profile (for 4 players) with the payoff:

\[
\begin{align*}
   b^*(1) &= (1,1) \quad \Rightarrow \quad \pi_1(1,1) = \frac{91}{512} \approx 0.18 \\
   b^*(2) &= (3,3) \quad \Rightarrow \quad \pi_2(3,3) = \frac{247}{653} \approx 0.38 \\
   b^*(3) &= (4,4) \quad \Rightarrow \quad \pi_3(4,4) = \frac{43}{653} \approx 0.63 \\
   b^*(4) &= (6,6) \quad \Rightarrow \quad \pi_4(6,6) = \frac{413}{1536} \approx 0.28 \\
   b^*(5) &= (7,7) \quad \Rightarrow \quad \pi_5(7,7) = \frac{1093}{1536} \approx 0.71 \\
   b^*(6) &= (9,9) \quad \Rightarrow \quad \pi_6(9,9) = \frac{559}{1536} \approx 0.36.
\end{align*}
\]

**Proof 10** It follows the same lines as the one for two players, but is quite messy. Therefore, we omit it here but it can be obtained from the author upon request.

And the expected payoff to a bidder is:

\[ E\pi_i^D = \frac{1}{6} \sum_{i=1}^{6} \pi_i = \frac{11}{32} \approx 0.34. \]
Multi-unit common value auctions: A laboratory experiment with three sealed-bid mechanisms

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Abstract
This study addresses a discrete common value environment with independent (one-dimensional) private signals, where the seller offers two identical units and the buyers have (flat) demand for both. Each session is conducted with 2, 3 or 4 buyers. Three auction formats are used: the discriminatory, uniform and Vickrey auctions which are all subjected to a variation in the number of bidders and to repeating bid rounds on each subject. The main findings are that there are no significant differences between the uniform and the discriminatory auction in collecting revenue, while the Vickrey auction comes out as inferior. More bidders in the auction result in a greater revenue and level out the performance across the mechanisms. Demand reduction is visible in the experiment, but it is not as prominent as anticipated. Moreover, subjects come closer to equilibrium play over time. Finally, the winner's curse is less severe than what is reported for inexperienced bidders in other studies.

Keywords: Laboratory Experiment; Multi-Unit Auction; Common Value Auction

JEL codes: C91; C72; D44

1 Introduction
Common value (CV) auctions with single unit demand have been studied for quite some time, both theoretically and in the laboratory. The main focus of the experimental research on CV auctions has been on the winner's curse problem, that is, the adverse selection effect produced by a win if not accounted for. But research on multi-unit demand is scarce. The winner's curse problem has not been addressed in this literature; the emphasis in both theoretical and experimental research, when the items for sale are substitutes,
Multi-unit common value auctions: A laboratory experiment with three sealed-bid mechanisms

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Abstract

This study addresses a discrete common value environment with independent (one-dimensional) private signals, where the seller offers two identical units and the buyers have (flat) demand for both. Each session is conducted with 2, 3 or 4 buyers. Three auction formats are used: the discriminatory, uniform and Vickrey auctions which are all subjected to a variation in the number of bidders and to repeating bid rounds on each subject. The main findings are that there are no significant differences between the uniform and the discriminatory auction in collecting revenue, while the Vickrey auction comes out as inferior. More bidders in the auction result in a greater revenue and level out the performance across the mechanisms. Demand reduction is visible in the experiment, but it is not as prominent as anticipated. Moreover, subjects come closer to equilibrium play over time. Finally, the winner’s curse is less severe than what is reported for inexperienced bidders in other studies.

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1 Introduction

Common value (CV) auctions with single unit demand have been studied for quite some time, both theoretically and in the laboratory. The main focus of the experimental research on CV auctions has been on the winner’s curse problem, that is, the adverse selection effect produced by a win if not accounted for. But research on multi-unit demand is scarce. The winner’s curse problem has not been addressed in this literature; the emphasis in both theoretical and experimental research, when the items for sale are substitutes,
has been on demand reduction. A phenomenon in some auction formats is that bidders have an incentive to reduce the demand for units other than the first, since these bids may become the market clearing price. It is found that demand reduction leads to substantial revenue losses for the sellers. There is also a literature concerning mechanism design issues, complementariness and synergies between items, and the role of package bidding (see, for example, Kagel and Levin (2011) for a start in these areas).

The prevalent static multi-unit auction formats in the literature are the discriminatory, the uniform, and the Vickrey auction. The first two formats are those used in the field, whereas the last is never used due to its (allegedly) complicated nature, even though it has nice demand-revealing properties; see, for example, Rothkopf et al. (1990). When we add the common value environment, the ranking of these auction formats in term of revenues becomes an open question. There is also an ongoing discussion in the market for treasury bonds, as well as in the markets for CO$_2$ allowances, on which of the first two formats above should be used. (Back and Zender (1993) summarize this debate in the independent private value (IPV) case.)

This study features a discrete auction, in the sense that the values of the unit and bidding are only allowed in integer numbers with independent (one-dimensional) private signals, where the seller offers two identical units and the buyers have demand for both. The three auction formats (discussed in the above paragraph) are tried and subjected to a variation in the number of bidders and to repeating bid rounds (15 - 20 rounds) on each subject. Five main questions are scrutinized. (i) which auction format gives the greatest revenue?; (ii) how does the number of bidders affect revenue?; (iii) is there demand reduction in the uniform and Vickrey auctions?; (iv) what are the implications of repeating the auction several rounds on the subjects, that is do we see any learning effects?; and (v), is there a winner's curse, that is do bidders ignore the informational content inherent in winning, and bid too high?

Starting with revenue, we find that the Vickrey auction always gives the least overall revenue, especially in small group sizes. The uniform and the discriminatory auctions run a close race and cannot be separated. This was quite unexpected due to the non-expected result in 2-player groups. (The hypothesis for the uniform auction is that, in 2-player groups, the subjects play more according to the extreme demand reduction prediction. But, in general, they do not.) For large group sizes, the difference in revenue between the Vickrey and the other two formats disappears completely. The answer to the second

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1 Even though Vickrey (1961) was the first to point out the inefficiency of multi-unit auctions in general, Ausubel (2004) and Ausubel and Crampton (2002) emphasized, in common value settings, that the inefficiencies are due to demand reduction.
question is that the more bidders in an auction, the larger is the revenue for the seller. Third, we see demand reduction, but we do not see any extreme demand reduction at all, that is, zero bidding on the second unit. Fourth, we find that subjects do learn to play equilibrium strategies in the course of the game, at least in the discriminatory auction. Moreover, they continue to learn until the final rounds.

For the last question, we find that the winner’s curse (WC) is highly present; mostly in the uniform and discriminatory auctions, but also in the Vickrey auction. We distinguish between bidding above the conditional expected value (of winning) up to the naive expected value and above the naive expected value. It is twice as common to bid in the first interval, which (partly) indicates that subjects have difficulties in understanding the winner’s curse.

The theoretical model in this article emanates from Ahlberg (2009) where it is presented more thoroughly. There is little earlier theoretical and experimental work on multi-unit demand common value auctions against which to directly compare our results, except for the theoretical article from Álvares and Mazón (2010). They have a theoretical model similar to ours in a continuous setting. Much of the theory that exists focuses on independent private value (IPV) settings, or, to some extent, interdependent value settings. The contribution of the present study to the experimental literature is a different common value generation, which is made somewhat simpler for ease of understanding. There is reason to believe that subjects do not understand the concept of the winner’s curse, and overbid as a result. Second, we want to contribute to the ongoing debate on which of the static auction formats one should use in practice, when the value of the object(s) is common to all bidders.

The rest of the paper is organized as follows: Section 2 focuses on earlier research in relation to the stated questions, section 3 presents the theory and the hypotheses, and section 4 outlines the experimental design. Section 5 contains the results, section 6 discusses them and section 7 concludes the study. All computations are found in the Appendix.

2 Earlier Research

In previous analytical research, Ausubel and Crampton (2002) have shown that, in the interdependent value case where the item for sale is infinitely divisible, in many cases, the discriminatory auction outperforms the uniform price auction but, in general, the revenue ranking between the two is ambiguous. In an IPV setting, Engelmann and Grimm (2009) investigate the three auction mechanisms described above and two open counterparts; the open uniform auction and the Ausubel auction, which is a dynamic Vickrey auc-
tion. They find that the revenues are greater if a sealed-bid format is used as compared to an open auction; the revenues depend less on which pricing rule is employed.

Kagel and Levin (2001) theoretically predict that as the number of bidders increases, demand reduction will diminish. They confirm this behavior asymmetrically in a laboratory experiment; that is, in which subjects behave according to theory only if their rivals decrease in number, not if they increase. Katzman (1995) also provides a theory indicating that the prevalence of demand reduction decreases with the number of participants even though some demand reduction will always be present. Engelbrecht-Wiggans et al. (2006) also establish that there is no difference in the first-unit-bid between the uniform auction and the Vickrey auction when the number of bidders increases from two to three (or five), even though the second-unit-bid is always greater in the Vickrey auction.

Concerning demand reduction, Noussair (1995) showed in a seminal paper that the bid on the second unit was always lower than its value, in contrast to the first bid, which was always demand-revealing. The degree of under-revelation depends on whether the bid sets the price or not. Ausubel and Crampton (2002) provide a formal proof of demand reduction in the uniform auction. Katzman (1995) and Engelbrecht-Wiggans and Kahn (1998) also analyze auctions that involve demand reduction.

Demand reduction has been confirmed in experimental research to various degrees, for example in a field experiment by List and Lucking-Reiley (2000) in which two players with two-unit demand bid for two units through the uniform price, the English, or the Vickrey auction, also replicated in a laboratory experiment by Porter and Vragov (2006). Another laboratory experiment where demand reduction is confirmed is by Kagel and Levin (2001); they let one bidder with two-unit demand compete against a robot bidder with unit demand and playing the dominant strategy.

With respect to learning, we have the evolutionary paradigm, or what Nelson and Winter (2002) call the "competence puzzle", which roughly means that individuals typically do not have the vast computational and cognitive powers that are imputed to them by the optimization-based theories (such as that in this article). But, since learning, guided by clear short-term feedback, can be remarkably powerful even in addressing complex challenges, the evolutionary response to the competence puzzle focuses on the role of learning and practice.

The research on the winner’s curse is vast, starting with Capen et al. (1971) who claimed that oil companies suffered from low returns. A comprehensive survey of theory and experiments in single-unit, common value auctions is offered by Kagel and Levin (2002). They show that the WC is pervasive across
various types of auctions and is not eliminated, only somewhat mitigated, by experience or even by using expert bidders. But experimental studies on common value, multi-unit auctions are scarce.

But one, notably, is Ausubel et al. (2009), which experimentally tests alternative auction designs suitable for pricing and removing troubled assets. They make use of the same static and dynamic uniform auction as this study and Engelmann and Grimm (2009) above, except that their dynamic format is an Ausubel descending clock auction. The units for sale are not identical, and they sell the units individually or as pooled units. And, for some sessions, bidders also know their liquidity needs. They find that the static and dynamic auctions resulted in similar prices. However, the dynamic auctions resulted in substantially higher bidders’ payoffs, which enabled the bidders to better manage their liquidity needs. The dynamic auction was also better in terms of price discovery, as well as for reducing the bidder error.

Another study is Manelli et al. (2006) which experimentally compares the static Vickrey auction with the Ausubel auction, also known as the dynamic Vickrey auction, in both an IPV setting and an interdependent value (IV) setting, in which the values are affiliated. They conclude that due to overbidding in both types of auctions, but slightly more in the Vickrey auction, the revenue from the Vickrey auction is greater, while the efficiency is lower in the Ausubel auction. But in the IV setting, they observe less overbidding and a trade-off between efficiency and revenue; the Vickrey auction is more efficient while the revenue is higher in the Ausubel auction.

3 Experimental Design

The experiment used students from KTH (the Royal Institute of Technology) as experimental subjects. They were from different Master’s programs in Engineering, and the experiment took place in May 2009. In total, 152 unique subjects participated in the experiment.

The subjects were recruited for computer sessions consisting of a series of auction periods. Each subject participated in one of three possible auction formats; hence, the design is between subjects. In each period, two identical units of a commodity were sold to the two highest bids, and these two bids could come from the same bidder or two different bidders. The units had no meaning for the participants apart from the money they could bring forth. Only the subject(s) who won the units earned profit(s), calculated as the induced value of the item minus the price paid for it. When there were ties, the winning bids were randomly selected.
The procedure for generating the common value was as follows: The value of the units for sale was generated by two random integers which were added together. Both units had the same value for the potential buyers, i.e. bidders had flat demand curves. To construct a tight market, we let the integers be chosen from the set \{1, 2, \ldots, 6\}. Thus, the possible values for each item were \{2, 3, \ldots, 12\}. The bidders were not fully informed about this value, though. Rather, each bidder was independently and randomly shown one of the two integers as a private signal. For expository reasons, we displayed these signals as dice. Subjects were told that two dice were rolled and added up as the common value, but each of them was only allowed to see one of the dice, independently of each other. Thus, the distribution of both the value and the signal was common knowledge. When the die was displayed on the screen, the subjects bid on both units. Only integer-value-bids between 0 and 12 were allowed. Subjects were also informed that the order of the bids was irrelevant. Bidder instructions are find in Appendix C.

Hence, a common value environment is created where private signals may be used to create unbiased estimates of the value of the items. If \( t_i \) is the signal, the common value will lie in \( V = t_i + \{1, 2, \ldots, 6\} \) and an unbiased estimator of the value (ex ante) is then \( \hat{v}_i = t_i + \frac{7}{2} \). Thus, the signals are positively correlated (affiliated) with the value. The underlying distribution of the private signals in the experiment was common knowledge; that is, everyone was told that she would see one of the two dice and her competitors might, but not necessarily, see the same die.

The group size was limited to two, three and four participants, respectively. Two approaches to allocating subjects to groups were used. In the first, each participant always competed against the same number of opponents, but not necessarily against the same opponents. Before each new round, all participants within each group size were re-randomized against each other. In the second approach, all participants were re-randomized against each other before each new round, irrespective of the group size. The reason for re-randomizing each new round was to counteract subject-specific effects and tacit collusion. Moreover, in advance of every new round, the common value of the last round was displayed on the screen alongside the two winning bids and the price paid for the two units. Moreover, to ensure that comparisons among auction formats were not driven by particular configurations of value, the two integers constituting the value were randomly generated for each new auction.

Each subject got SEK 100 as a participation fee, or show-up fee, and a starting balance of SEK 50 to cover losses. Profit and losses were added to this balance. If a participant’s balance went negative, he or she was suspended from the auction and had to leave (with the participation fee). The others were paid in cash at the end of the experiment.
One of the justifications for the starting balance is that, even if participants play the risk-neutral Nash equilibrium, losses may occur. A starting balance also imposes opportunity costs for overly aggressive bidding, and is enough for errors made during bidding and a reasonable return for participating if aggressive bidders shut them out of the auction.

4 Theory and Hypotheses

The theoretical model in this article originates from Ahlberg (2009) where it is presented more thoroughly. An excerpt from it is available in Appendix A.

Starting with the revenue question, and beginning with 2-player groups, equation 9 in the Appendix shows there to be a unique equilibrium strategy in the discriminatory auction; it prescribes the player to bid equal amounts on both units. For the uniform and Vickrey auctions, there is no unique strategy; the uniform auction has a multitude of equilibria, whereas the Vickrey auction has dualistic equilibria. One thing in common for both, however, is that they have extreme demand reduction equilibria, i.e. equilibria that prescribe a zero bid on the second unit and thereby transform the players into single-unit demanders. Equations 11 and 13 in the Appendix also show these to be the payoff-dominating equilibria. The equations show that the equilibrium bid for the first unit is to bid the conditional expected value, which makes the equilibrium risk dominate the other equilibria. (Bidders could also bid above this value for the first unit, but with a higher risk.)

Thus, using the unique strategy in the discriminatory auction and the payoff-dominated equilibria in the two other auctions, the following expected revenues for a two-player game emerges (see eq. 14 and 15):

\[ E[R^D(2 \text{ Players})] = 11.22 \]
\[ E[R^{U,V}(2 \text{ Players})] = 0 \]

where \( U \) stands for the uniform, \( D \) for the discriminatory and \( V \) for the Vickrey pricing rule.

Since the ex ante expected value for the two units for sale is 14, we see that, in the discriminatory auction, the seller captures the major part of the value at stake, but zero in the two other formats. Thus, the discriminatory auction gets the highest ranking. As regards the two other formats, the payoff-dominating equilibrium for the uniform auction is somewhat more robust (in the uniqueness of the bidder’s best response) than the payoff-dominating equilibrium of the Vickrey auction (see section 9.5.3), which thus indicates that the Vickrey
When there are more than two players in the auctions, the extreme demand-reduction strategy of bidding zero on the second unit disappears. (Since it is always an equilibrium to bid the conditional expected value on the first unit, notwithstanding the group-size, the price-setting bid will never be zero; thus, a zero-bid on the second unit does not gain anything.) However, the bidders must now be cautious about the first unit bid, as it can be the price-setting bid. Following the comparison of conjectures 7 and 8 in Appendix B for the uniform auction with the strategies for the discriminatory auction in section 9.5.1, we have that the uniform auction always gives a greater revenue than the discriminatory auction when there are 3 or 4 players. Hence,

\[
E[R^U(3 \text{ Players})] > E[R^D(3 \text{ Players})] \]
\[
E[R^U(4 \text{ Players})] > E[R^D(4 \text{ Players})].
\]

In the Vickrey auction, all pure equilibria disappear due to non-core outcomes since there is always a coalition for which the total payoff becomes higher; notwithstanding the individual strategy. Thus, we can formulate the following hypothesis:

**Hypothesis 1 (Revenue Comparison)** If there are only two bidders in the auction, the discriminatory auction will outperform the two other mechanisms, and the Vickrey auction will have a marginally higher rank than the uniform auction. With more bidders, the uniform auction is likely to give more revenue than the discriminatory auction.

The second topic is how the number of bidders influences the bidding and hence, the revenue. The expected revenue is calculated using the unique strategies for the three group sizes of the discriminatory auction (see eq. 14 in the Appendix):

\[
E[R^D(2 \text{ Players})] = 11.22 \quad (1)
\]
\[
E[R^D(3 \text{ Players})] = 12.38 \quad (2)
\]
\[
E[R^D(4 \text{ Players})] = 12.63. \quad (3)
\]

If we start with a two-player game and increase the number of bidders by one, the revenue increases by almost 10 percent. If we go from the three to the four-player game, the revenue increase is just 2 percent. Since going to five players only increases the revenue by 1.5 percent, we also see that using four players actually captures the idea of “many” bidders. Thus, in this setting, the expected revenue seems to increase concavely with the number of bidders.
The same is also true for the uniform auction, because of diminishing incentives for demand reduction the more players there are. From conjectures 7 and 8, we have that both the bids on the first and the second units (weakly) increase with the number of participants. Thus, in terms of expected revenue, we have

\[ E[R^U(2 \text{ Players})] > E[R^U(3 \text{ Players})] > E[R^U(4 \text{ Players})]. \]

Going from two to three players, we have a clear revenue ranking following the disappearance of the extreme demand reduction. The next step is less pronounced, but at least there is a distinct difference in revenue.

Thereby, we have the following hypothesis:

**Hypothesis 2 (The number of bidders)** *The revenue will increase with the number of bidders. Moreover, we expect to see a greater difference between two and three-player groups as compared to three and four-player groups. This should be true for both the discriminatory and the uniform auctions.*

A third hypothesis concerns the uniform and the Vickrey auctions. We have seen (above) that, when there are only two bidders, there could be zero bids on the second unit. This, in turn, gives zero revenue to the seller. Hence,

**Hypothesis 3 (Demand reduction)** *When there are two bidders, bids on the second unit will be (much) lower than the expected value (i.e. under-revealing) in the uniform and Vickrey auctions.*

The fourth hypothesis concerns profit maximization and learning, i.e., evolutionary aspects of the bidding process. Each settled round gives feedback on the performance which, correctly interpreted, gives an indication of how to bid in the next round. So even if subjects do not understand how to compute an equilibrium strategy, they may roughly learn a rule of thumb.

Thus, an iterative process may be needed to approach equilibrium play.

**Hypothesis 4 (Learning)** *Subjects are likely to use strategies closer to (theoretical) equilibrium play over time.*

Last, the winner’s curse (WC) is scrutinized. The first to recognize the WC was Capen et al. (1971) who argued that the low rates of return among oil companies in the 1960s and 1970s on OCS lease sales, year after year, resulted from bidders’ ignorance about the informational consequences of winning.

Hence, we define the WC as the adverse selection effect of bidders neglecting the information a win will produce. That is, that the announcement of winning leads to a decrease in the estimated value of the objects, if not accounted for when bidding (given a symmetric game and that the high signal holder(s)
win(s) the objects). The underlying cause in this study is that, even though the signal plus the expected value (EV) of the other integer is an unbiased estimator of the value, the max operator of all $t_i + 7/2$ is not; it is a convex function and thus overestimates the value.

In the present design, the lower bound estimate of the value is $(t_i + 1)$ and the upper bound is $(t_i + 6)$. The strategy of bidding the risk-free lower bound strategy never yields a negative payoff, whereas bidding above the upper bound would ensure a negative payoff. The unbiased, though naive, EV of the items is $E(v|t_i) = (t_i + 7/2)$. It is naive in the sense that it is the EV, independent of winning the item(s). Define $E(v|t_i > t_{-i})$ as the EV, for player $i$, conditional on having the highest signal, $t_i$. Since the informational content of winning leads to a decrease in the estimated value, and cannot be lower than the lower bound, it must lie in the interval $\{t_i + 1, t_i + 7/2\}$. For $t_i \in \{1, 2, \ldots, 6\}$ it is $E(v|t_i > t_{-i}) = 3t_i/2$.

A bidder who does not take this fully into account and uses the naive EV instead of the appropriate EV conditional of winning when placing her bids could, upon winning, pay more than the estimated value of the object(s). The systematic failure to account for this is referred to as the winner’s curse.

The difference $E(v|t_i) - E(v|t_i > t_{-i})$ decreases with the signal $t_i$; meaning that the greater the signal, the less the bidder has to shade the bid to account for the winner’s curse. Or, stated differently, it is worse to find out that one won with a low signal rather than a high.

In the present analysis, we discriminate between bidding in the WC interval, which s then above $E(v|t_i > t_{-i})$ up to the naive $E(v|t_i)$, and bidding above the latter. And since it is risk-free to bid $(t_i + 1)$, the interval in question, i.e. the WC interval, becomes $\{t_i + 2, t_i + 3\}$. The reason for the division of the intervals is that, theoretically, bidding above the naive EV has nothing to do with the WC. Bidding above the naive EV will, on average, produce a negative profit. But since this discrimination is not made in other experiments, we will also pool the result.

\[\text{Table 1}\]

<table>
<thead>
<tr>
<th>No. of subjects</th>
<th>No. of rounds</th>
<th>Unique observations</th>
<th>$\bar{\text{c}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\text{No. of subjects No. of rounds Unique observations $\bar{\text{c}}$}\]

\[\text{2 That is to say, bidding in the discriminatory auction. For the uniform and Vickrey auctions, where players do not pay what they bid, the word bidding should be interpreted as paying.}\]

\[\text{3 Bidding above the unbiased estimate $t_i + 7/2$ is not rational, ex ante. But, there is also what Holt and Sherman (1994) call a loser’s curse. The expected value of the item, conditional on not winning, is greater than the naive expected value. In this model it is, for bidder $i$, $E(v|t_i < t_{-i}) = \frac{3t_i + 2}{2} > t_i + 7/2 = E(v|t_i)$. The difference $E(v|t_i < t_{-i}) - E(v|t_i)$ increases with the signal $t_i$, meaning that the greater the signal, the more the bidder might have bid to account for the loser’s curse. By inspection of the data, we conclude that in this experiment, the loser’s curse is non-existent.}\]
Moreover, our way of generating the signal from the CV makes the CV upwardly biased from the signal. That is to say, even though \( \hat{v}_i = t_i + 2 \) is unbiased, \( t_i \) underestimates the CV. In, for example, Kagel et al. (1987), where the signal is drawn from a set consisting of \( \pm c \) of the CV, there is a certain region where the signal by itself is an unbiased estimator of the CV. We believe that the latter method will produce more WC due to the fact that in fifty percent of the draws, the signal is below the CV.

There is vast experimental evidence of the WC for both inexperienced players and professionals in single-unit auctions, see Kagel (1995). Due to the inherent demand reduction equilibria in two of the auction formats, the WC should be lower on the second unit for sale. Thus, we conclude that:

**Hypothesis 5 (Winner’s curse)** *The winner’s curse will be apparent, but not so much as reported in other experiments since the common value in the present experiment is biased upwards. And the WC should be considerably smaller for the second unit, as compared to the first.*

### 5 Experimental Results

We conducted 15 or 20 rounds of bidding for each subject; the number was stochastically determined, not known in advance by the subjects (they did not know how many rounds they were going to play). The data description is in Table 1, which shows, for each format, the number of subjects, how many rounds there were, the number of unique observations, and the average profit. Each format consisted of groups of two, three and four bidders.

<table>
<thead>
<tr>
<th>Format</th>
<th>No. of subjects</th>
<th>No. of rounds</th>
<th>Unique observations</th>
<th>( \bar{\pi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discriminatory</td>
<td>62</td>
<td>345</td>
<td>941</td>
<td>0.37</td>
</tr>
<tr>
<td>Uniform</td>
<td>44</td>
<td>256</td>
<td>745</td>
<td>0.36</td>
</tr>
<tr>
<td>Vickrey</td>
<td>46</td>
<td>286</td>
<td>777</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 1

There were two experimental designs; one configuration where subjects remained in the same group size in all rounds, i.e. the number of competitors was always constant for them, but the competitors changed; and another where subjects were randomized without any constraints in all rounds, i.e. both the number of individual competitors and the competitors changed. But, when using the highest (or lowest) bid as the dependent variable in an ordinary least squares regression, these different designs do not have any significant influence. Nor when the design interacts with rounds, auction format or group
size is there a significant effect on the highest (or lowest) bid. Therefore, the data from the two experimental designs is pooled.

We use a first unit bid and a high bid interchangeably, meaning the (weakly) highest bid of the two bids that each subject submits. Likewise, a second unit bid and a low bid refer to the (weakly) lower bid of the two. The non-parametric Wilcoxon(-Mann-Whitney) rank sum test is used for comparing data between treatments, if not stated differently. There seems to be no problem with dependencies within subjects, nor within groups. We have made tests with OLS and Panel data (random effects) models with revenue (price) as the dependent variable. Revenue is explained by format, group-size, round and design. We have also made interactions between group-size and format on the above. Moreover, we have used both the difference in bids and equilibrium bids as dependent variables, explained by the same covariates as the former, and interactions between group-size and format. The below presented results only changed marginally and, thus, the conclusions still hold. (The OLS regression on revenue can be found in Appendix B.)

Last, in the discriminatory auction, roughly 11 percent of the subjects went bankrupt. For the uniform auction, that portion was only about 4 percent, whereas the Vickrey auction had zero bankruptcies.

Table 2 displays the average revenue for different auction formats (rows) and group sizes (columns). The numbers inside the brackets are the revenues in Bayesian equilibrium, to the extent that it is found. (See eq 1 for the discriminatory auction. The uniform and Vickrey auctions are the payoff dominating Bayesian equilibrium, that is, the extreme demand reduction equilibrium.)

<table>
<thead>
<tr>
<th>Group Size</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>11.70 (0)</td>
<td>12.89</td>
<td>14.69</td>
<td>12.97</td>
</tr>
<tr>
<td>Vickrey</td>
<td>9.49 (0)</td>
<td>11.65</td>
<td>14.59</td>
<td>11.22</td>
</tr>
</tbody>
</table>

Table 2
Average revenue, with predicted revenue inside the parenthesis.

**Hypothesis 1:**

Overall, there are no significant differences between the discriminatory and the uniform auction as concerns concerning revenue. The Vickrey format is inferior, especially for small group sizes. Interestingly enough, the larger the group size, the closer the revenues are between the auctions. Looking at 4-groups alone, the formats are not significantly different from each other. In the other group-sizes, and when groups are pooled, the Vickrey auction collects significantly less revenue than both other formats ($p$-values $< 0.01$).
Result 1 (Revenue Comparison) In 2-player groups, the discriminatory auction together with the uniform auction collects significantly more revenue than the Vickrey auction. This is contrary to the hypothesis that the discriminatory auction should outperform the uniform auction. When there were more bidders, the hypothesis was that the uniform auction should give a weakly higher revenue than the discriminatory auction; which did not happen either.

Hypothesis 2:

Table 2 also indicates that larger auction groups give more revenue. All p-values except one are below 0.01; the one above is 0.1156 and concerns the discriminatory auction between 3- and 4-groups.

Result 2 (The number of bidders) The result for both the uniform and the Vickrey auctions supports the hypothesis that the revenue increases with the number of bidders. For the discriminatory auction, the result is not as strong because of the high significance level (15-percent level) between two group sizes; but the result, therefore, verifies that the revenue increase is concave in that format.

Hypothesis 3:

Demand reduction, or bid shading, means that bidders do not apply the demand-revealing strategies. In the discriminatory auction, the unique symmetric strategy for bidders is to bid equally on both units. Hence, there should not be any demand reduction in that format. However, in both the uniform and the Vickrey auctions, there are optimal strategies that are both demand revealing and not. In the latter strategies, the bidders will always bid below the expected value for the second unit, possibly zero. We will also report demand reduction for the discriminatory auction.

For all three formats, first, Table 3 shows the frequency of the bid-spread and, second, the value of the bid-spread, third, given that the bids are not equal, what is the frequency for Bid 1 to be above the EV and, finally, given that the bids are not equal, with what frequency Bid 2 is below the EV. In other words, the next to last column shows if the subjects engaged in a bid-spread overbid or not for the first unit, and, in the last column, if they underbid or not for the second unit.

<table>
<thead>
<tr>
<th></th>
<th>Bid-spread</th>
<th>Bid-spread value</th>
<th>Bid 1 &gt; EV</th>
<th>Bid 2 &lt; EV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discriminatory</td>
<td>0.58</td>
<td>0.88</td>
<td>0.15</td>
<td>0.90</td>
</tr>
<tr>
<td>Uniform</td>
<td>0.78</td>
<td>1.85</td>
<td>0.55</td>
<td>0.70</td>
</tr>
<tr>
<td>Vickrey</td>
<td>0.73</td>
<td>1.41</td>
<td>0.50</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 3
Frequency and value of bid-spread.
The uniform and the Vickrey auction have overlapping strategies, given the same signal, even though the price rule differs between them. The discriminatory auction has both different strategies, given the same signal, and a different pricing rule. This explains the similar frequencies in the last two formats as well as the great discrepancy between them and those of the discriminatory auction.

Hypothesis 3 suggests that there should be, if not complete, at least a great under-revelation for the second bid in both uniform and Vickrey auctions. First, not seen in the table, we conclude that there is very little extreme demand reduction behavior, that is, zero bids on the second unit, even though this is the payoff-dominating strategy in games with two players for the uniform and Vickrey auctions (only 5 percent of the bids in the uniform auction, and 3 percent in the two other formats). There is no significant difference in zero-bids between the formats.

If we use the bid-spread as a measure for demand reduction, we see that the uniform and the Vickrey auction have quite the same frequency of demand reduction; whereas the discriminatory auction has a lower frequency. Nevertheless, all three formats are significantly different from each other (p-values < 0.022). Comparing the values, there is a larger spread between the formats; which is a reflection of the significant difference between them (p-values < 0.001).

The next-to-last column tells us that, for the uniform and Vickrey auction, half of the subjects who engage in demand reduction also bid above the value on the first unit bid. Thus, we must look at the second unit bid to understand if demand reduction is present. (It could be the case that both bids are above value, which would then not really be demand reduction.) This cannot be compared to the much lower frequency in the discriminatory auction, where the winning bids become the price. In the last column, we see the frequency of under-revealing bids. All bids should be under revealing in the discriminatory auction, which is almost the case. More interesting, both the uniform and the Vickery auction have about seventy percent second unit bids below the expected value.\textsuperscript{4} The two formats do not differ significantly from each other in this respect.

**Result 3 (Demand reduction)** We find evidence of demand reduction on the second unit but, in contrast to hypothesis 3, very few zero bids on the second

\textsuperscript{4} The 70% share of bid 2 below EV almost coincides with the Porter and Vragov (2006) result. They had a 68% share in an IPV experiment with two bidders and two units for sale. But they just got a 30% share for the Vickrey auction. (One must be cautious when making a comparison with their results since they have a different value system to the one in this study.) Moreover, we are now using the EV, and not the EV conditional of winning, since we are looking at all subjects.
unit. All formats differ significantly from each other on both the frequency and the value of the bid-spread. And since the uniform and Vickrey auctions do not differ in the under revealing of the second unit bid frequency, we conclude that the uniform auction produces more demand reduction than the Vickrey auction. As for the discriminatory auction, where the symmetric equilibrium does not predict demand reduction, the format has significantly lower values and frequency compared with the other two.

Hypothesis 4:

For the discriminatory auction, we measure learning as the share of first and second unit bids, consistent with the theoretical, extended-equilibrium strategy where the extended-equilibrium strategy is defined as: \( b_1, b_2 \in (b^* - 1, b^* + 1) \). Does the share of bids in this interval increase with the number of rounds played?

Equipped with this definition, a learning effect in the discriminatory auction can be seen in Table 4. Moreover, this effect is concave. Between rounds 4 – 6 and the middle rounds, it is significant at the one-percent level, and between the middle rounds and the last three rounds, it is significant at the five-percent level.

<table>
<thead>
<tr>
<th>Round</th>
<th>4 – 6</th>
<th>11 – 13</th>
<th>18 – 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discriminatory</td>
<td>61 %</td>
<td>75 %</td>
<td>85 %</td>
</tr>
</tbody>
</table>

Table 4
Frequencies of (extended) optimal bids

Regarding the uniform and the Vickrey auctions, we do not have any equilibrium strategy prediction for more than two players. Thus, we concentrate on 2-player groups, and measure learning as finding the payoff-dominating equilibrium strategy. Hence, do subjects increase the number of zero-bids on the second unit along with the rounds played, or, at least lower the second unit bid as the session continues?

There was no such effect in the Vickrey auction; whereas there was a tendency to it in the uniform auction. That is to say, the p-value was 0.1125 when comparing the second unit of the early rounds with the last three rounds.

Result 4 (Learning) In the discriminatory auction, the subjects moved towards the optimal strategy over time, consistent with hypothesis 4. The subjects also continued to learn in later periods, but to a lesser extent. That is, the learning effect is concave (at least between the measuring points). In the uniform auction, the learning was barely significant, but the subjects seemed to weakly understand the demand-reduction equilibria over time (rounds).

Comment:
There is especially one odd result here when compared to theory, which spurs both anomalies in hypotheses 1 and 3. The subjects’ bids in 2-player groups were expected to be (much) lower in the uniform and the Vickrey auctions. Even if the revenue does not go to zero as predicted by theory, it should at least be much lower (than the discriminatory auction). Maybe the competitive element, or the joy of winning, overtook any rationale in these groups. Even though some subjects understood having to play zero on the second unit and high on the first, their opponents seldom did. Another prominent feature in the experiment is the low revenue outcome in the Vickrey auction for two- and three-player games. Supposedly because of its complicated nature, the subjects did not seem to understand this.

**Hypothesis 5:**

As described above, the WC interval is defined as bids above the EV conditional on winning, $EV_i = E(v|t_i > t_{-i}) = 3t_i/2$, up to the naive EV, $EV_n = E(v|t_i) = (t_i + 7/2)$. Since it may not be intuitive to grasp the underlying cause of the winner’s curse, i.e. the convexity of the max function, bids in this interval could be rationalized on the basis of the fact that they are (at least) below the naive expected value. However, bids above $E(v|t_i)$ are, on average, never individual-rational since they produce a negative profit on average. Moreover, to separate the random component from the actual bid, we distinguish between bidding in the WC interval and actually experiencing a negative profit, i.e. suffering from the winner’s curse, in Tables 4 to 6 below.

In the uniform and Vickrey auctions, the bid is just a proxy for the price, since subjects do (often) not pay what they bid. The price-setting bid could come from any bidder in the uniform auction but, in the Vickrey auction, it is always another player’s bid that becomes the price-setting bid. Thus, in these two formats, we are measuring more like a generalized WC; a WC within each group. All bid frequencies, or prices, in table 5 are conditional on both winning and having the high signal. That is, as stated in the above paragraph, even though we are using bid 1 and bid 2 in the table, it is the price that these bids generated that we are measuring for the last two formats.

The table is to be interpreted as follows: In the discriminatory auction, 35 percent of the second-unit bids were in the WC interval. Of these, 57 percent de facto gave a negative profit. Thus, a total of 20 percent second-unit bids gave negative profits.

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5 Cox et al. (1992) tried to explain overbidding in IPV first price auctions with ‘joy of winning’ and Cooper and Hamming (2008) partly support a modified version of the ‘joy of winning’ hypothesis in an experimental study of the IPV second-price auction.

6 It could be individual-rational for 2-player groups in the uniform and Vickrey auctions. But that hinges on how the other player bids, and it is still quite risky.
average. on average, never individual-rational since they produce a negative profit on EV auctions. But that hinges on how the other player bids, and it is still quite risky. 

It could be individual-rational for 2-player groups in the uniform and Vickrey auctions. As described above, the WC interval is defined as bids above the EV condition: 

$$\begin{align*}
EV_n \geq b_1 > EV_c \\
\text{Winner's curse} \\
\text{Total} \\
\hline
b_1 & b_2 & b_1 & b_2 & b_1 & b_2 \\
\hline
\text{Discriminatory} & 0.42 & 0.35 & 0.42 & 0.57 & 0.18 & 0.20 \\
\text{Uniform} & 0.37 & 0.32 & 0.47 & 0.42 & 0.17 & 0.13 \\
\text{Vickrey} & 0.36 & 0.30 & 0.34 & 0.33 & 0.12 & 0.10 \\
\end{align*}$$

Table 5
The frequency of winner’s curse bids and actually experienced winner’s curse.

At a first glance, there is no significant difference between the first and second unit prices within each auction type (the p-values starting from 0.1229 and rising.). But looking more closely reveals a pattern due to the lower frequency of second unit bids/prices vs. first unit bid/prices in all three formats. (There is an interval ranging from 0.05 to 0.07 between the two bids.) This is also confirmed with a p-value of 0.0545, when testing all formats together for differences between the two bids. Hence, we have a distinct difference between bids for subjects when pooling all auction formats.

Switching to a comparison between auction formats, and once more testing for differences between bids in the WC-interval, we only find a difference between the discriminatory and Vickrey auctions; the p-value is 0.0928 when comparing first unit prices.

Since the difference between the first and second unit bids is weak in this case, Table 8 shows the results when pooling first and second unit bids. It can

$$\begin{align*}
EV_n \geq b_1 > EV_c \\
\text{Winner's curse} \\
\text{Total} \\
\hline
b_1 & b_2 & b_1 & b_2 & b_1 & b_2 \\
\hline
\text{Discriminatory} & 0.40 & 0.46 & 0.18 \\
\text{Uniform} & 0.36 & 0.46 & 0.17 \\
\text{Vickrey} & 0.34 & 0.34 & 0.12 \\
\end{align*}$$

Table 6
The frequency of winner’s curse bids and experienced winner’s curse, when bids 1 & 2 are pooled.

be seen that the uniform and the discriminatory auctions are almost identical when analyzing the total WC. The Vickrey auction has almost the same frequency of bids in the WC interval, but experienced WC is lower; hence, it has a roughly 30 percent lower total WC when compared with the other two. As above, the only difference in bids between the auctions is between the discriminatory and Vickrey auctions; now the p-value becomes somewhat
lower, namely 0.0692, when the first and second unit prices are pooled. Still, the significance levels are quite weak.

When bids/prices above the expected value are scrutinized which, as mentioned above, is not really a WC problem but shown here for reference, the following table emerges (Table 7). Only the bid for the second unit in the discriminatory auction differs significantly from the others when analyzing bids above \( EV_n \). Thus, the pooled results \( (b_1 \& b_2) \) are also shown in the table, as is the total WC. (To be comparable with the analysis above, all bid frequencies, or prices, in the table are conditional on both winning and having the high signal.)

As for the discriminatory auction, subjects are more cautious when bidding on the second unit, and the lion’s share of the second unit bids give a negative payoff, but the analysis of the second unit bids is to be taken with caution due to lack of data. The lack of data on the second-unit bids is shown in both columns of pooled bids, since the pooled results highly resemble the first unit-bids. But, in total, the formats are not significantly different from each other in both bidding above expected value and making a negative profit.

The result of pooling all bids above \( EV_c \) is shown in Table 8. The formats are not significantly different from each other, so the ranking is ambiguous.

<table>
<thead>
<tr>
<th>( b_i &gt; EV_n )</th>
<th>Negative profit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>( b_2 )</td>
<td>( b_1 &amp; b_2 )</td>
</tr>
<tr>
<td>Discriminatory</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>Uniform</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>Vickrey</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 7
Frequency of bids above the (naive) expected value, and the negative payoff.

Table 8
The percentage of bids giving negative profits, in total
Result 5 (Winner’s curse) The winner’s curse is highly visible, but does not have as large an effect on outcomes as in results from earlier experiments, c.f. Kagel and Levin (2002), for inexperienced bidders in single unit auctions. The cases of WC are also robust across the sample population and not just for a couple of bidders. Across group sizes, there is no difference between the sizes in bidding in the WC interval.

Comment:

As stated at the beginning, for players in 2-player groups, it could be an equilibrium strategy to bid on or above any of the expected values. But this is only the case if both bidders bid zero on the second unit, which never happened. Thus, bids from 2-player groups are included in the above results.

The impact-differences of the winner’s curse across distinct set-ups, i.e. other experiments, could be explained from the construction of the common value interval and the private signal generation. Here, if a player got a 6 (1) as a signal, he/she knew that the signal was the highest (lowest) possible. This is, of course, valuable information. One purpose of this set-up was to make the idea of common value clear and uncomplicated to understand and, thus, the winner’s curse would be mitigated. This was the case in the present experiment. Moreover, due to demand reduction, the WC would be lower in multi-unit settings than in single-unit settings.

6 Discussion

First, we notice that as the number of players increases, the pricing rules converge in collecting revenue. When there were only two bidders in the auction, all formats were significantly different in revenue raising, but when there were four bidders, the difference became insignificant. Thus, attracting bidders, or ensuring competition, could be much more important than selecting the auction form.

There was one particularly odd result in the experiment, namely the high revenue for 2-player groups in the uniform auction. This was rather unexpected because of the anticipated low revenue equilibria outcome of this group. One possible explanation is the competitive element; subjects did not play the theoretical equilibrium at all; but wanted to win the object(s), no matter what the costs. Holt and Sherman (1994) explain this as the joy of winning phenomenon in their study. In the present study, it was encountered not only in this particular group size, but was pretty common in all group sizes in all auction formats.
As for the winner’s curse, we chose to solely isolate the WC interval. Many experiments do not distinguish between the intervals and, thus, treat all bids above the EV conditional on winning as potential winner’s curse bids, which, per definition, they are not. But, of course, all bids above the EV conditional of winning are dangerous and could give rise to a negative profit. Thus, we present the results from the bids above the naive EV, and then the experiment is comparable with the results from other experiments. Another issue concerning the WC was that it entailed no learning effects; subjects continued to suffer from the winner’s curse in later rounds, and not just in the early rounds. They never really grasped the idea.

Contrary to the non-learning in the WC problem, there was another type of learning that we chose to discuss here because of lack of evidence in the data. Subjects learned in the course of play, i.e. they adapted to what the other player(s) did in the auction and bid according to that. In other words, they were trying to find a best-response function. Nonetheless, because of the common value structure, where the random component played a part of the profit earned, it is hard to see the evidence in the data.

All subjects were inexperienced players, and one must be careful in drawing policy recommendations from the result. But other research, Kagel and Levin (2002) for example, has shown that overbidding is a robust feature, not only for bidders with no experience, but also for professionals.

7 Conclusion

The present paper has studied the two most common auction formats used in the field, the discriminatory and the uniform auctions, as well as the Vickrey auction, a more theoretical format. All three formats make use of two treatments; first, varying the number of bidders and, second, repeating the auction several times inside each session.

The main conclusion was that the auction format is less important for revenue generation when the number of bidders is large; there were no significant differences in the revenue of the three formats when there were four competitors in the auction. Neither of the discriminatory and the uniform auctions could be distinguished as better at revenue generation than the other; only the Vickrey auction could be classified as inferior, compared to the others, when there were few bidders. One possible explanation for this could be the complicated nature of the Vickrey auction, which subjects had difficulties in understanding.

Interestingly enough in the experiment, almost no one understood the extreme
demand-reduction equilibrium. Very few, indeed, grasped the idea of bidding zero when there were just two bidders in the uniform and the Vickrey auctions. Moreover, in the discriminatory auction, subjects learned to play equilibrium strategy over time.

The WC is still a problem; overall, subjects did not seem to understand the adverse selection effect that winning produces. Regardless of group size or auction type, the WC was always there for about 17 percent for the two most common auctions, and around 12 percent for the Vickrey auction. The WC in this study is defined as bids/prices between the expected value conditional on winning and the (usual, naive) expected value, not bids above this expected value. Bids above the naive expected value were somewhat less common, and quite the same in all three formats, around 9 percent.

8 Acknowledgements

We would like to thank Lars Hultkrantz, Jan-Eric Nilsson and Svante Mandell for valuable comments on the paper. Also, thanks to Jan-Erik Swärdh for important help with econometrics. This study has been conducted within the Centre for Transport Studies (CTS). The author is responsible for any remaining errors.

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9 Appendix A

The theoretical model is found in Ahlberg (2009), but an excerpt is presented here.

9.1 The Model

In a Bayesian game, players update their prior beliefs by Bayes’ rule, as well as the opponents’ payoff functions, once they learn their types.

9.2 Posterior beliefs

Let the type-vector of all players except player \( i \) be denoted by \( t_{-i} = (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n) \). Then, given a player’s own type \( t_i \), denote \( \mu(t_{-i} | t_i) \) as player \( i \)'s conditional probability, or posterior beliefs, about her opponents’ types.

\[
\mu(t_{-i} | t_i) = \begin{cases} 
\frac{1}{2^n - 1} \cdot \frac{1}{6} & \text{all types in } t_{-i} \text{ are equal to } t_i, \\
\binom{n-1}{x} \cdot \frac{1}{2^n - 1} \cdot \frac{1}{6} & \text{At least one type in } t_{-1} \text{ is different from } t_i, \\
\frac{1}{2^n - 1} \cdot \frac{1}{6} & \text{all types in } t_{-i} \text{ are different from } t_i.
\end{cases}
\]

where \( x \) is defined as the number of players who are of the same type as player \( i \). The first equation works as follows: If all other players, except player \( i \), see the same value as \( t_i \), two things can happen. Either they all see the same die as player \( i \), which happens with probability \( \frac{1}{2^n - 1} \cdot \frac{1}{6} \), or at least one of the others sees a different die with the same value as \( t_i \), which happens with probability \( \frac{2^{n-1} - 1}{2^n - 1} \cdot \frac{1}{6} \). (The first term is the probability that at least one sees a different die and the second term is the probability that the die has the same value as \( t_i \).)

The second equation says that if one of the non-\( i \) players sees a different value than player \( i \), the belief for player \( i \) becomes \( \binom{n-1}{x} \cdot \frac{1}{2^n - 1} \cdot \frac{1}{6} \). In the last row, all non-\( i \) players see a different value than player \( i \), and, since we only have two dice, they all see the same die. This happens with probability \( \frac{1}{2^n - 1} \cdot \frac{1}{6} \).

9.3 Strategy and payoff

The strategy for each player is to assign two (integer-)bids, one for each item, from her signal. Formally, the strategy for player \( i \) is a mapping from her signal space \( T_i = \{1, 2, \ldots, 6\} \) to the two-dimensional space of integers, \( b_i : T_i \to \mathbb{Z}_+^2 \), where \( b_i(t_i) = (b_{i,1}, b_{i,2}) \).
Let $D_1, D_2$ be the random variables describing the outcome of the two integers, or dice, respectively. Then, the value for each bidder is the realization of the two variables, hence $v = d_1 + d_2$. Define $k_i(b)$ as the number of items won by player $i$ if strategy profile $b$ is employed.

Now, let $b = (b_1, \ldots, b_n)$ be a strategy profile and let $v = d_1 + d_2$ be a random variable. Then, player $i$’s payoff function $\pi_i : b \rightarrow \mathbb{R}$ is defined as

$$\pi_i(b, v) = k_i(b)v - p_i(b).$$

Thus, the payoff is the number of items won multiplied by the realized value of these items, minus the price the winner has to pay for them.

### 9.4 Expected value functions

Since players only get to see one integer, i.e., their signal, they have to use the expected value when calculating their value, which is $v(t_i) = t_i + 7/2$, that is, the value of her signal plus the expected value of the other die. But in a Bayesian game they also need to calculate their competitors’ value, given their own signal. This conditional expected value for the other players is dependent on how many players there are in the game/auction.

The fact that induces this is that they can all see the same integer or different integers. The more signals (players), the more accurate becomes the conditional expected value. That is, with many bidders, we approach the true value. This is an application of information aggregation, studied by Wilson (1977). The conditional expected value is defined as:

$$v_i(t_{-i}|t_i) = v(t_{-i}|t_i)$$

$$= \begin{cases} 
\frac{1}{2^{n-1}}(t_i + 7) + \frac{2^{n-1}-1}{2^{n-1}} \cdot 2t_i & \text{all } t_j = t_i, \\
\frac{2^{n-1}-1}{2^{n-1}}t_i + \frac{1}{2^{n-1}}7 & \text{all } t_j = t_i, \\
\frac{1}{2^{n-1}}(t_i + 7) + \frac{2^{n-1}-1}{2^{n-1}} \cdot 2t_i & \text{all } t_j = t_i.
\end{cases}$$

The first row in equation (5) says that if the non-$i$ players are of the same type, $t_i$, as player $i$, two things can happen. Either they see the same die as player $i$, which occurs with probability $\frac{1}{2^{n-1}}$, or at least one of them sees a different die. The value for the former becomes $t_i + 7/2$ for player $i$, while the value for the latter becomes $t_i + t_i = 2t_i$.

In the second row of the same equation, we see the value if one, or both, is of a different type than player $i$. Then, since there are only two distinct integers, the value becomes the sum of the integer values. Equation (6) is just a simplification.
The term expectation above means expectation over the possible outcomes of the integer values. From player $i$’s perspective, if we also take expectations over all $t_{-i}$, we get the expected value for a competitor, given player $i$’s type. That is, we must combine the posterior beliefs with the conditional expected value to get the expected value for any competitor. Hence, the expected value for a competitor to player $i$ is defined as

$$E_{t_{-i}}[v(t_{-i}|t_i)] = \sum_{t_{-i}} \mu(t_{-i}|t_i)v(t_{-i}|t_i).$$

If we also take the expectation over all $t_i$, the terms will sum up to seven as they should. But for any given $t_i$, the expected value for a competitor will not be $t_i + 7/2$. This is the case since we have to take into account that the competitor may get the signal from the same die as player $i$.

### 9.5 Equilibrium

A *Bayesian equilibrium* of this game with a finite number of types $t_i$, for each player $i$, and a common prior distribution $\mu$, and pure strategy spaces $T_i$ is a Nash equilibrium of the ”expanded game” where each player $i$’s space of pure strategies is the set $(\mathbb{Z}_k^2)^{T_i}$ of maps from $T_i$ to $\mathbb{Z}_k^2$.

Given strategy profile $b(\cdot)$, and $b'_i(\cdot) \in (\mathbb{Z}_k^2)^{T_i}$, let $(b'_i(\cdot), b_{-i}(\cdot))$ denote the profile where player $i$ plays $b'_i(\cdot)$ and the other players follow $b(\cdot)$, and let

$$(b'_i(t_i), b_{-i}(t_{-i})) = (b_1(t_1), \ldots, b_{i-1}(t_{i-1}), b'_i(t_i), b_{i+1}(t_{i+1}), \ldots, b_n(t_n))$$

denote the value of this profile at $(t_i, t_{-i})$. Then, since all types have positive probabilities, the bid/strategy $b_i(t_i)$ is a (pure strategy) Bayesian equilibrium if player $i$ maximizes her expected utility conditional on $t_i$ for each $t_{-i}$:

$$b_i(t_i) \in \arg\max_{b'_i \in \mathbb{Z}_k^2} \sum_{t_{-i}} \mu(t_{-i}|t_i)[k_i(b'_i, b_{-i})v(t_{-i}|t_i) - p_i^f(b'_i, b_{-i})].$$

We only allow integer-value-bids\(^7\). Since the value function is symmetrical and we have a symmetrical joint distribution, only types will be of importance when bidding; thus, we look for a symmetrical equilibrium.

### 9.5.1 The Discriminatory auction

In this auction, conditional on winning, for each item won, every bidder pays the price of her bid on that item.

---

\(^7\) A pure strategy Bayesian equilibrium in $\mathbb{R}$ does not exist.
From equation 8 in the Appendix A, we can derive the following unique pure Bayesian equilibrium strategy for two bidders:

\[ b^*(t_i) = (t_i + \left\lfloor \frac{t_i}{3} \right\rfloor, t_i + \left\lfloor \frac{t_i}{3} \right\rfloor), \]  

(9)

where \( \lfloor x \rfloor \) is a ceiling function which maps \( x \) to the smallest following integer, i.e. \( \lfloor x \rfloor = \min\{n \in \mathbb{Z}|n \geq x\} \). The striking feature is that players bid the same amount on both units.\(^8\)

When we increase the bidders by one, all bidders but the type-6 player bid the same as in a two-player game. The type-6 players raise their bids on both units by one increment unit.

If we look at the four-player game, we get the same increase in the bids for the type-6 players as in the three-player game, but a reduction in the bids for the type-1 players. The reduction is one increment. The optimal strategy when there are four players can then be written as:

\[ b^*(t_i) = (t_i + \left\lfloor \frac{t_i}{2} \right\rfloor, t_i + \left\lfloor \frac{t_i}{2} \right\rfloor), \]  

(10)

where \( \lfloor x \rfloor \) is a floor function which maps \( x \) to the largest previous integer, i.e. \( \lfloor x \rfloor = \max\{m \in \mathbb{Z}|m \leq x\} \).

9.5.2 Uniform auction

**Conjecture 6 (Two players)** In a two-player game, no other equilibrium payoff dominates the following:

\[ b^*(t_i) = (b^*_1, b^*_2) = (\lceil v(t_j|t_i) \rceil, 0), \]  

(11)

where \( \lceil x \rceil \) is the nearest integer to \( x \) upwardly.

**Proof 1** Suppose that player \( j \) utilizes \( b^*(t_j) \). Any attempt to win 2 units for player \( i \) would make her second unit bid set the price. And since the bid from player \( j \) is \( b^*_1(t_i) = \lceil v(t_j|1) \rceil = 4 \), player \( i \) must bid at least 5 to win. The payoff for using \( b^*(t_i) \) is the expected value minus the price paid, which is zero, hence \( \pi^* = t_i + 7/2 \), while the expected value for using the alternative strategy would be \( \pi' \leq 2(t_i + 7/2 - 5) \). Then, we have that \( \pi' > \pi^* \) implies (at best) \( 2(t_i + 7/2 - 5) > (t_i + 7/2) \Rightarrow t_i > 6 \), which is impossible.

As a matter of fact, when there are two bidders, any bid above \( \lceil v(t_j|t_i) \rceil \) on the first unit is an equilibrium bid. In an IPV setting, Levin (2005) has shown that any

\(^8\) Lebrun and Tremblay (2003) give a more general proof of this result when values are private.
bid weakly above the upper endpoints of the distribution, if the reservation price is zero, is an equilibrium. This is indeed true also in this model, but the proposed equilibrium risk dominates all other equilibria.

But there also exist other equilibria. If both bidders bid 1, 2 or 3 on the second unit, irrespective of \( t_i \), the bids also become equilibrium bids. But since it is highly unclear on which of these equilibria the subjects would coordinate, the zero-bid on the second unit is focal as well as payoff-dominating in undominated strategies.

When there are more than two players in the game, two things happen. First, we have to correct downwards instead of upwards, as above, because now there is a chance that someone’s first-unit bid may become the price-setting bid. And, second, as a result of the first, the zero bid on the second unit is no longer an equilibrium. This is the case since there are now at least three bidders and two units, and all three bidders have a weak incentive to bid the true (expected) value of the first unit.

**Conjecture 7 (More than two players)** When there are more than two bidders in the auction, it is an undominated strategy to bid the following on the first unit:

\[
b^*_i(t_i) = \lfloor v(t_j|t_i) \rfloor.
\]

where \( \lfloor x \rfloor \) is defined as the nearest integer of \( x \) downwardly.

**Proof 2** First, note that to bid more than \( b^*_i \) will incur an expected loss if the bid is above both \( \lfloor v(t_j|t_i) \rfloor \) and the price. That is, suppose that player \( i \) bids \( b^*_i > \lfloor v(t_j|t_i) \rfloor \). Then if \( b^*_i > p > \lfloor v(t_j|t_i) \rfloor \), a loss of \( p - \lfloor v(t_j|t_i) \rfloor \) will be realized on that unit.

Second, suppose that the bid is below the equilibrium bid \( b^*_1 < b^*_i \). Then, three cases appear; first, if the bid is below the value which, in turn, is weakly below the price, i.e. \( p \geq \lfloor v(t_j|t_i) \rfloor > b^*_1 \), then nothing would change if the player were to raise the bid to \( \lfloor v(t_j|t_i) \rfloor \). Next, if the bid is below \( \lfloor v(t_j|t_i) \rfloor \) and above the price, \( \lfloor v(t_j|t_i) \rfloor > b^*_i > p \), nothing would change here either if the bid was increased to \( \lfloor v(t_j|t_i) \rfloor \). The last case is if the value is greater than the price and the price is weakly greater than the bid, \( \lfloor v(t_j|t_i) \rfloor > p \geq b^*_1 \). Now, if the player raised the bid to \( \lfloor v(t_j|t_i) \rfloor \), she would win a unit at a more profitable price. Thus, to bid the proposed equilibrium bid on the first unit is (weakly) dominant in expectation.

Now, by the last conjecture, when there are more than two players, the bid on the second unit will be weakly bounded from below by the first-unit bid from the low type player. Thus

**Conjecture 8 (More than two players)** The second unit bid is weakly bounded by 4, i.e. \( b^*_2(t_i) \geq b^*_1(1) = 4 \).

**Proof 3** If player \( i \), say, bids below 4, she will win at most one unit and get the payoff: \( \pi^*_i = (t_i + 7/2 - p)k^*_i \), where \( k^*_i \leq 1 \). If the player bids 4, the payoff will be: \( \pi^*_i = (t_i + 7/2 - p)k^*_i \), where \( k^*_i \geq k^*_1 \) since the bid \( b^*_2(t_i) \) now competes against the
other bids, which the zero bid did not. And, since the bid does not affect the price, p will be the same in both payoff functions above. Hence, $\pi^*_i \geq \pi'_i$.

**Conjecture 9 (Many players)** The more bidders in the auction, the higher the bids. This is true for both the first-unit bid and the second-unit bid.

**Proof 4** Given any realization of the two dice, we see from equation (6) that the conditional expected value weakly increases with the number of players. Besides, as can be seen from conjectures 7 and 8, since both the first-unit bid and the second-unit bid are dependent on that value, we have that both bids increase with the number of players.

### 9.5.3 The Vickrey auction

In the Vickrey auction, a player who wins $k_i$ units pays the $k_i$ highest losing bids of the other players - that is, the $k_i$ highest losing bids not including her own. Hence, the winner is asked to pay an amount equal to the externality she exerts on other competing bidders.

The Vickrey auction is known to have an *ex post equilibrium*, or a *no-regret equilibrium*. That is, an ex post equilibrium is a Bayesian equilibrium with the additional requirement that even if all players’ signals were known to a particular bidder, it would still be optimal for her not to alter her strategy, that is, she would not suffer from any regret. This Bayesian strategy is:

$$b^*(t_i) = (t_i + \lceil \frac{t_i}{2} \rceil + 2, t_i + \lceil \frac{t_i}{2} \rceil + 1),$$

where $\lceil \cdot \rceil$ is defined as the nearest integer upwardly. This is indeed an equilibrium:

**Proof 5** If the type-$t_i$ bidder bids less, the number of units that she wins is at most what she would win by bidding $b^*(t_i)$. For any of the units won, the prices will be the same as before, but she will forgo some surplus for units that she did not win.

If she instead bids $b(t_i) > b^*(t_i)$, then she wins at least as many units as before. The prices for the first $k_i$ units will remain the same as if she bid $b^*(t_i)$. For any additional units, however, the price paid will be too high, since for $k > k_i$, the price is greater than the value for the item(s).

But, as in the uniform auction, we have an extreme demand reduction strategy. This equilibrium is the same as in the uniform auction, i.e.:$$b^*(t_i) = \lceil v(t_i) \rceil), 0). \quad (13)$$

---

9 For this to be true in this setting, we must have that the value function satisfies what Ausubel (1999) calls *value monotonicity* and *value regularity*. This, indeed, is true for the value function in this paper.
Proof 6 The proof is as in the uniform auction, hence it is omitted.

But this demand reduction strategy is a much weaker equilibrium strategy in the Vickrey auction than in the uniform auction, because, if player $i$ bids the above strategy in the uniform auction, player $j$’s best response is to bid the same. That is not entirely true in the Vickrey auction since you never pay what you bid, but what the other bids. Hence, in the Vickrey auction, player $j$ can bid any number below her conditional expected value for the second unit and still be an equilibrium strategy. And, by the same token, any bid below the conditional expected value is an equilibrium bid. For this equilibrium, as in the uniform auction, we have that any bid on the first unit above the conditional expected value is an equilibrium bid.

9.6 Expected revenue

In a (pure) common value auction, revenue is strongly negatively correlated with profit. And seen above, both the uniform and the Vickrey auctions have equilibria that give the entire surplus to the buyer, which is the same as the expected value of the two integers, i.e. 7. This translates into zero revenue to the seller.

The discriminatory auction, on the other hand, has a unique equilibrium, and to find the expected revenue, we calculate the probability for each set of possible joint signals between the players. Then, we make use of the strategies implicitly inherent in the signals to compute the price paid for each possible set of joint signals. Then, we have the expected revenue as the product of the intersection of the signals times the realized price in that outcome. For two players, player $i$ and player $j$, it becomes:

$$E[R] = P(t_i \cap t_j) p(b_i, b_j),$$

where $p(b_i, b_j)$ is the price paid. If there are three players, we instead calculate $P(t_i \cap t_j \cap t_k) p(b_i, b_j, b_k)$, and so on.

By doing this computation, we have for the discriminatory auction:

$$E[R^D(2 \text{ Players})] = 11.22$$
$$E[R^D(3 \text{ Players})] = 12.38$$
$$E[R^D(4 \text{ Players})] = 12.63. \quad (14)$$

As we already have stated, both the uniform and the Vickrey auction give zero revenue:

$$E[R^{U,V}(2 \text{ Players})] = 0 \quad (15)$$
ESSAY II

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$$E[R] = P(t_i \cap t_j) p(b_i, b_j)$$

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$$E[R_{U,V}(2 \text{ Players})] = 0$$

Table 9

Regression on revenue (price)

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS</th>
<th>Robust standard error</th>
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</thead>
<tbody>
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<td>Reference</td>
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</tr>
<tr>
<td>3-player groups</td>
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<td>0.31</td>
</tr>
<tr>
<td>4-player groups</td>
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<td>0.33</td>
</tr>
<tr>
<td>Vickrey auction</td>
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<tr>
<td>Uniform auction</td>
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<td>Design 1</td>
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</tr>
<tr>
<td>Intercept</td>
<td>11.40***</td>
<td>0.28</td>
</tr>
</tbody>
</table>

No of observations: 766

$R^2$: 0.181

Notes: a; Dependent variable is revenue (price).
b; ***, ** and * denote difference from zero at the one, five and ten percent significance level respectively.

Table 9
Regression on revenue (price)
11 Appendix C

Bidder instructions for the uniform, common value auction:

11.1 Introduction

Hello and welcome. You will participate in an experiment on economic decision-making. The purpose is to study sales by bidding, i.e. through an auction.

You have the opportunity to win money through participation. The show-up fee is SEK 100 (€10), and by learning the rules of the game you have the opportunity to earn more than that. On the other hand, you could also lose in the process. To ensure that you walk away with at least SEK 100 in your pocket, we give you a starting balance of SEK 50. If you lose this money, you will be excluded from the experiment. Your winnings, and the show-up fee, will be paid in cash after the experiment.

A rule that applies at all times is that all communication between participants is prohibited. If you have any questions, raise your hand and I will come to you and you may ask your question in a whisper. If I believe the question must be answered, I will repeat it to everyone and give the answer.

11.2 Design

Rounds: The experiment consists of several rounds. In each round, 2 identical objects, or units, are to be sold through an auction. (How many rounds to actually be played will be unknown to you.)

The commodities: We will name the units as unit A and unit B. Each of you has a value associated with owning these units and would like to buy them. We call this the redemption value, which is the same for both units.

The redemption value: Before the start of each round, the value of the units is randomly determined through the roll of two dice. The redemption value will then be the sum of the dice. The value can thus never be less than 2, and the maximum is 12. Therefore, the (value) v belongs to the set \( \{2, 3, \ldots, 11, 12\} \). However, you will not know what this value is. Instead, you will get private information about this value.

Information: Your information will consist of one of the dice; the other die will be hidden. Thus, you have to make your bids with only partial information of the value. The program randomizes which of the two dice you will see. Other players may, but must not, see the same die as you do.

Note: If you do not have one of the highest bids, nothing happens. The profit is zero.
Opponents: Before the beginning of each round, the program will randomly choose how many players you will be matched with. You can have one, two or three opponents. Your group-size will be seen on your screen.

Bids: After receiving your information, that is, after seeing your die, you should decide on what you want to bid for the units. You are permitted to place equal or different bids for the units.

11.3 Instructions

Buy: Those who have placed the highest bid, and the next-to-highest bid, purchase the units. This may be the same person or two different people. If there are ties among the (winning) bids, the program will randomly choose the winner(s).

Price: The winners will pay a price equal to the highest bid that does not win, That is, the highest bid that is rejected. All winners pay the same price for the units.

Example: 2 units are sold. Three people (A, B, C) have the three highest bids: 10 (A), 9 (B), 8 (C). A and B purchase the units, and both pay 8.

Gain/Loss: The winners make a profit equal to the difference between the (re-demption) value and the price. If the difference is negative, a loss is the result.

Example of profit: You won one unit, and the price was 6. The value of the unit was 8. You made a profit of 2 ($8 - 6 = 2$).

Example of a loss: You won one unit, and the price was 10. The value of the unit was 8. You then made a loss of 2 ($8 - 10 = -2$).

Note If you do not have one of the highest bids, nothing happens. The profit is zero.

11.4 Practical execution

Bidding: You will come to a (web-)page where you see two dice, one of them without dots. The one with dots is your signal. Below the dice, there will be 2 fields, one for unit A and one for unit B. You place your bids for the two units in these fields. Only integers between 0 and 12 are possible. (The units are identical, and each bid is for one of the two units.)

Money: You will see what your current balance is before every game starts on the screen. The starting balance is 10 experimental currency. These will be converted to SEK 5/1 at the end of the experiment. If you lose your starting balance, the auction is over for you.

Lost starting balance: If someone (or some) loses her starting balance, she will no longer participate in the auction. This means that there will be one (or more) person(s) less in the auction. If that happens, the auction continues as usual
without them but, since we need to have even groups, the program randomizes which players are going to play in subsequent rounds. You may have to pass a round or two. You will be given notice about that on your screen.

**One round:** After you enter your bids in the fields, press the button "Add bids". When everyone has pressed the button, bids are ranked. Those who have placed the highest bids purchase units at a price that is determined by the pricing rule for each auction.

If there are more winning bids than units for sale, the program randomizes the winners. The balance is recalculated and a new round starts. On the screen you will see the redemption value for the units, the price, the winning bids, own won units, and own profits/losses.

**The end:** After a certain number of rounds, the experiment will end and you will come to a page showing what you have earned in the experiment.

11.5 **Summary**

- You will play a certain number of rounds and, in each round, two identical units are for sale.
- You will play against one, two or three opponents. On the screen you will see the number of opponents you have in the current round.
- In each round, all players in an auction have the same redemption value for both units.
- Each player only gets an informational signal about the true value. Subjects may or may not see the same information as their opponents.
- You place two bids, one for each unit. You are allowed to place equal or different bids on the units.
- You start with 10 experimental currency. If you lose this, the experiment is finished for you, and you are excluded from the experiment. But you can also earn more, depending how you and your opponents act.
**ESSAY II**

**Bidder Instructions for the Discriminatory and Vickrey Auctions:**

The item price in the Instructions above is changed for the two other auction formats; for the discriminatory auction it is:

**Price:** The winners pay a price equal to their own placed bid.

*Example:* 2 units are sold. Three people (A, B, C) have the three highest bids: 10 (A), 9 (B), 8 (C). A and B purchase the units, and they pay 10 and 9, respectively.

And for the Vickrey auction, we have:

**Price:** The winner(s) pays a price equal to the highest bid that does not win, not including his own. That is, the highest bid that is rejected and comes from someone else.

*Example 1:* 2 units are sold. Four people (A, B, C, D) have the four highest bids: 10 (A), 9 (B), 8 (C) and 7 (D). A and B purchase the units, and both pay 8.

*Example 2:* 2 units are sold. Three people (A, B, C) have the four highest bids: 11 (A), 10 (B₁), 9 (B₂), 8 (C). A and B purchase one unit each, A pays 9, and B pays 8 (since 9 is his bid).

*Example 3:* 2 units are sold. Three people (A, B, C) have the four highest bids: 7 (A₁), 6 (A₂), 5 (B), 4 (C). A purchases both units; for the first he pays 5 and for the second he pays 4.

Otherwise, the instructions for the formats are the same.
Multi-unit common value auctions: An experimental comparison between the static and the dynamic uniform auction

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Abstract
It is still an open question whether the dynamic or the static format should be used in multi-unit settings, in a uniform price auction. The present study conducts an economic experiment in a common value environment, where it is found that it is more a question of whether the auctioneer wants to facilitate price discovery, and thereby lessen the otherwise pervasive overbidding, or if only the revenue is important. The experiment in the present paper provides evidence that the static format gives a significantly greater revenue than the dynamic auction, in both small and large group sizes. But a higher revenue comes at a cost; half of the auctions in the static format yield negative profits to the bidders, the winner's curse is more severely widespread in the static auction, and only a minority of the bidders use the equilibrium bidding strategy.

Keywords: Laboratory Experiment; Multi-Unit Auction; Common Value Auction

JEL codes: C91; C72; D44

1 Introduction
In many auctions, such as for CO$_2$ allowances, electricity, bonds, etc, the auctioneer wants to sell many items at the same time, and bidders are usually not content with buying just one unit. All units for sale have the same value for bidders in these auctions. That is, the profit is linear, or is a multiple of the number of units won. For some of them, as a first approximation, the value is also equal across bidders because the value of the unit often depends on...
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1 Introduction

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some outside parameter, common to all bidders. Such auctions are referred to as common value auctions. Even though the valuation of the items across bidders is identical in a common value auction, it is unknown at the time of bidding. Bidders’ information only consists of a (privately known) signal.

When there are secondary markets, as in the emission permits market or the bond market, the price in the secondary market can be a good estimator of the price in the auction; that is, a common price signal for all bidders. But, private signals also exist. For example, when there are market-dominant participants in the auction, they could, due to their (demand) size, be price drivers. This is especially true when there is a fixed quantity for sale, and big participants need/want a large share of the supply. Then, their own demand is one type of private signal. Signals can also contain information such as (under hand) political information about change of rules, or technology changes which are not known to every participant in the auction.

Comparing the CO₂ allowance auctions in the USA and the EU, that is the Regional Greenhouse Gas Initiative (RGGI) and European Union Emission Trading Scheme (EU ETS), respectively, a striking difference is that the clearing price in the EU ETS is more than five times higher than in the RGGI. This discrepancy has a couple of different explanations; the fixed quantity, i.e. the shortage of units for sale, and the number of bidders, given the same supply. The two are correlated, and they are intertwined with the bidders’ demand. The quantity cap in the RGGI has been non-binding, the reserve price has been met in the last six auctions, whereas, for the EU ETS, the clearing price fluctuates with the number of bidders; the more bidders, the higher the price.

In the present study, we try to replicate the two allowance auctions mentioned, but without varying the cap (i.e. supply). The experiment makes use of two group sizes, the first includes a large group of bidders that more or less has the same relation between demand and supply as the EU ETS, and, second, a smaller group of bidders that has half the demand (and players) of the first group. For each group size, there are also two different demand sizes. Even though it is theoretically not the same to cut the number of bidders’ demands in half as to increase the supply twofold, there is experimental evidence that, contrary to the predictions of a Nash equilibrium, bidding does not decrease in response to an increased number of bidders. See Kagel (1995) and Ahlberg (2011).

1 For the CO₂ allowance auction case, the value is a proxy for the social abatement cost; in the electricity auction, the value comes from the electricity price; whereas in the bond case, the value is driven by the interest rate.

2 Data from RGGI can be found at http://www.rggi.org/market/market_monitor and data from the EU ETS can be found at http://ec.europa.eu/clima/policies/ets/auctioning/second/index_en.htm.
The two allowance auctions above employ a static, sealed bid, uniform price auction. Both theoretical and experimental economic research suggests that a dynamic auction format is preferable to the static auction in conducting allowance auctions. In the theoretical literature, Milgrom and Weber (1982) show, in single-unit, affiliated value auctions, that the informational content in open auctions reduces the bidder’s uncertainty about the (affiliated) value and thus, bidders are able to bid more aggressively in them. In the experimental literature, Kagel et al. (1987) report a pervasive bidding above value in the single-unit, IPV static auctions, which is, however, alleviated in the dynamic format. See also Kagel (1995).

In the common value (CV) environment, an essential advantage of dynamic, or open, bidding is that the bidding process reveals information about the other bidders’ estimates of the value. Consequently, the winner’s curse is likely to be mitigated in the open auction. The argument is that, by using tentative price information, bidders are better able to make more precise calculations about the value; thus the open auction facilitates price discovery.

The seminal closed-form equilibrium analysis of the winner’s curse (WC) was made by Wilson (1969), and has since then been shown by Bazerman and Samuelson (1983) in various experimental environments. In the present experiment, we discriminate between bidding above the conditional expected value (of winning) and the more naïve conventional expected value. The rationale is that bidding above the naïve expected value has nothing, strictly speaking, to do with the WC; it will transmit negative profit in the mean. Whereas bidding in the WC interval, which is defined as bidding in between the two expected values and winning, could ensure a negative profit; it depends on how other players bid.

Both these arguments run in favor of open bidding, rather than sealed bidding. The open paradigm is also widely used by the Federal Communication Commission when selling radio frequencies in the USA. In IPV settings, some research, e.g. Klemperer (2002) and Engelmann and Grimm (2009), instead calls for caution due to the facilitative facilitating effect of the open format on collusion between bidders, since all bids, or quantities demanded, are visible for all participants still in the auction.

Multi-unit, common value experiments are rare, and the experiment in the present study contributes to the ongoing debate on open or sealed bid auction mechanisms inside the uniform price mechanism. One exception is the closely related experiment conducted by Ausubel et al. (2009), which is focused on troubled assets and liquidity needs. They find that, even though the formats rendered similar prices, the open format gave substantially higher (bidder) payoffs as well as reduced bid errors.
This study compares two different uniform price auctions; the static and the ascending clock auction, both in a common value environment. To address the above questions, both formats are used in two group sizes: 3- and 6-bidder groups. Letting the configuration of the larger groups (in own demand) be exactly two times that of the smaller groups, and letting the supply be equal in both groups, is effectively comparing a loose and a tight cap at the same time (if bidding does not adapt to the increasing number of bidders). The loose cap, represented by the 3-player groups, has the relation \( \frac{1}{2} \) of supply (numerator) and aggregated demand (denominator), whereas the tight cap, or 6-player groups, has the relation \( \frac{2}{3} = \frac{1}{4} \). Moreover, the two group sizes always have the relation \( \frac{1}{2} \) between a large demander (numerator) and a small demander (denominator). The tight cap resembles the EU ETS auctions conducted in Great Britain (but which are open to participants throughout the EU).

The main results from the experiments are;

- The seller revenue is significantly greater in the sealed-bid format. But it comes at the cost of a considerably more negative profit for buyers, and nearly half of the auctions ended with a negative profit for the subjects.
- In line with this is the considerably smaller amount of WC in the open format, both bidding in the WC interval and experiencing a negative profit. There is also a notable quantity of bids above the conventional, naive, expected value, especially in the static format.
- The more bidders (the tighter the market), the greater the revenue.
- None of the formats seem to result in high bids that coincide with individual rationality. That is, there is overbidding; less than 1/5 of all subjects’ first unit bid/dropout is at, or below, the expected value of the unit.
- The demand reduction, measured as the bid spread, is visible in both formats, but it is significantly lower in the dynamic auction.

We conclude that the dynamic auction seems to be a better choice in common value environments, especially if the players are without experience. It facilitates price discovery, and thereby alleviates the overly aggressive bidding. The choice between an open or a closed format may be more important than the choice of price mechanism, especially in common value settings.

The remainder of the paper is organized as follows. Section 2 provides an overview of some earlier research, section 3 introduces the experimental model and delivers the hypotheses. Section 4 presents the experimental results, while section 5 discusses them. Section 6 concludes the paper.
2 Earlier research on static vs. dynamic formats

The research on multi-unit, common value auctions is still embryonic. Much has been done in the independent private value (IPV) field, especially with single unit demand. Vickrey (1961) was the first to show that, in theory, the static second-price auction produced efficient outcomes in the IPV setting with single unit demand. Vickrey was also the first to state the revenue equivalence theorem that under certain conditions, any allocation mechanism will lead to the same revenue for the seller. Riley and Sammelson (1981) and Myerson (1981) then generalized the theorem. In contrast to this, laboratory experiments have proved that the dynamic second-price auction, the English auction, performs roughly as predicted by theory, whereas the static second-price auction does not. One rationale for that is that the transparency of the dynamic mechanism guides subjects; see for example Kagel (1995).

In the multi-unit case, the seminal (game theoretic) article is by Wilson (1979), who, in an auction of shares, found collusive equilibria with prices lower than if the unit was sold as an indivisible unit. Later, Ausubel and Crampton (2002) showed that the efficiency of the second-price, multi-unit auction may break down due to demand reduction. Demand reduction, which is the phenomenon of bidders reducing demand (on marginal units) in favor of a lower market-clearing price, has been shown in a number of experiments since then. In analyzing the difference between the static and the dynamic uniform auction in a model with two bidders, with two-unit demand, Engelmann and Grimm (2009) see a larger share of demand reduction, especially extreme demand reduction, in the dynamic format versus the static in an IPV setting. Consistent with that, they also find that the static version outperforms the dynamic version in terms of collecting revenues as well as efficiency. Alsemgeest et al. (1998) also report lower revenues in the English clock auction as compared to the static version, due to demand reduction.

Vickrey (1961) also described an efficient mechanism in multi-unit settings in the IPV environment, nowadays called the Vickrey auction. Ausubel (2004) then came up with an open format that implements the same outcome as the multi-unit Vickrey auction in an IPV setting, and continues to be efficient in an affiliated value (AV) environment which is not the static Vickrey auction. Manelli et al. (2006) experimentally compare the static Vickrey auction with the Ausubel auction, also known as the dynamic Vickrey auction, in both an IPV setting and an interdependent value (IV) setting, where the values are

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3 The uniform auction is a generalized second-price auction, meaning that the price paid is the highest losing bid.
4 Affiliated value comes from Milgrom and Weber (1982), and roughly means that a high value of one bidder’s estimate makes high values of the others’ estimates more likely.
affiliated. They conclude that due to overbidding in both types of auctions, but slightly more in the Vickrey auction, the revenue from the Vickrey auction is greater, while the efficiency is lower in the Ausubel auction. But in the IV setting, they observe less overbidding and a trade-off between efficiency and revenue; the Vickrey auction is more efficient while the revenue is higher in the Ausubel auction.

Concerning the (pure) common value environment, much of the focus is on the winner’s curse and very few studies focus on the multi-unit case. One, notably, is Ausubel et al. (2009), which experimentally tests alternative auction designs suitable for pricing and removing troubled assets. They make use of the same static and dynamic uniform auction as the present study and Engelmann and Grimm (2009) above, except that their dynamic format is an Ausubel descending clock auction. The units for sale are not identical, and they sell the units individually or as pooled units. And, for some sessions, bidders also know their liquidity needs. They find that the static and dynamic auctions resulted in similar prices. However, the dynamic auctions resulted in substantially higher bidder payoffs, which made it possible for the bidders to better manage their liquidity needs. The dynamic auction was also better in terms of price discovery, as well as at reducing bidder error.

3 The Model and Hypotheses

The experiment will take place inside a multi-unit, common value (CV) auction model where bidders have independent (private) signals. Four units will be sold in each round, and all bidders place the same value v on each unit in a given auction, i.e. subjects have flat demand curves. The common value v is an integer drawn from a uniform distribution on the interval V = {10, 90}, and the signal s_i is uniformly distributed around this value, and lies in the interval S = {v − 10, v + 10} ⊆ {0, ..., 100}, for i ∈ {1, 2, 3} (3-player groups) or i ∈ {1, ..., 6} (6-player groups).

This method for generating values comes from Kagel et al. (1987), where it is used in a single unit CV auction and can be contrasted to that used in Manelli et al. (2006). In the latter study, all bidders get different private information about the value, and the CV is calculated as the weighted average of all bidders’ information. Thus, it is not a pure CV environment but an interdependent value environment. Ausubel et al. (2009) use different methods for generating values. In the first method, they let the CV for a security (or unit) be the average of eight iid random variables, uniformly distributed between 0 and 100, where a bidder’s private information about the unit is the realization of one of the random variables. In the second method, the high-value (U[50, 100]) and the low value (U[0, 50]) random variables are grouped
together in a pooled-unit auction. This is a pure CV environment but with non identical units.

One important implication of our (Kagel’s) way of generating the common value is the three distinct signal regions to which it gives rise, with a different informational content. The most interesting region, which encompasses the larger mass of bids, lies in \( \{20, 80\} \). (It is called region 2.) In this region, the signal is always an unbiased estimator of the true value, ex ante. The other two regions, regions 1 and 3, contain signals in the interval \( s_i \in \{0, ..., 19\} \) and \( s_i \in \{81, ..., 100\} \). The information that the signal is in one of these regions can be used to compute a more exact expected value than signals from region 2. That is, in region 1 (region 3), the signal is a downward (upward) biased estimator of the true value. And the lower (higher) the signal is in region 1 (region 3), the more downward (upward) biased it is. Signals at the endpoints can be used to compute an exact value.

Given signal \( s_i \), the estimated valuation will be contained in \( v_i \in [\max\{s_i - 10, 10\}, \min\{s_i + 10, 90\}] \). Bidders can place a risk free bid by bidding the lower end-point in this interval.

Two group sizes are used; 3-player groups and 6-player groups. As hypothesized, the two treatment groups can be seen as either representing a loose and a tight cap, respectively, or just plainly as two different group sizes. Inside the smaller group, one bidder demands 4 units and two bidders 2 units each. The larger group has the same relationship between small and large demanders, that is two 4-unit demanders and four 2-unit demanders. Aggregated demand is thus 8 (16) in 3-player (6-player) groups. The supply in each auction is 4 units. Thus, we have the relationship \( \frac{1}{2} \left( \frac{2}{3} \right) \) between supply and aggregated demand in small (large) groups.

In the static, the players bid in prices, whereas the dynamic is a quantity auction. In this quantity auction, the price is raised by means of a *price clock* and players respond with the quantities desired at the prevailing price. The quantity is restricted by an activity rule requiring monotonicity in quantities demanded, i.e. a dropout is irrevocable.

In the sealed bid auction, all bidders submit, once and for all, their bids, and then the auctioneer ranks the bids from high to low. The four highest bids are deemed to be winning bids, and the owners of these bids pay the fifth highest bid \( (b^5) \) for each unit won. (When there are ties, the winning bids are randomly determined.) Thus, if \( k_i \) is the number of units won in the auction for bidder \( i \), \( b_i \in \{0, ..., 100\}^l \) is the vector of bids for bidder \( i \) (where \( l \in \{2, 4\} \) is the the demand), and \( B \) is the downward ranked vector of all bids. Then, the profit for each bidder is \( \pi_i = k_i(v - B(5)) \).

The dynamic ascending auction is a natural generalization of the English
auction when selling more than one unit. In this auction, the price is gradually raised by means of an integer price-clock from zero to one hundred, and players start with full demand and yield units as the price rises. The auction ends when there are only four units demanded left in the auction, and all winners pay the price that prevailed when the fifth-to-last unit was surrendered. Thus, if \( P(5) \) is defined as the price that prevailed when the fifth-to-last unit in the auction was surrendered, the profit-function is similar to the one above \( \pi_i = k_i(v - P(5)) \), but now the bid \( B(5) \) has been changed to the clock-price \( P(5) \).

In an IPV auction, with only one unit for sale, \( B(2) \) and \( P(2) \) would have the same value, if the distribution of the values and signals were continuous, by the revenue equivalence theorem. But we have two extensions from this: First, this is a common value auction and, second, there are four units for sale. Regarding the first extension, Milgrom and Weber (1982) showed that the dynamic auction is always at least as good for revenue as the static counterpart in a CV auction. But in the multi-unit case, the ranking is less clear, especially with CVs. From Vickrey (1961), we have that all weakly non-dominated equilibria have one thing in common; namely, that the bid/dropout on the first unit should be the expected valuation of the unit. (The first unit means the unit with the weakly highest bid/dropout.) For the subsequent units, the theory is still vague.

Thus, even though the dynamic auction is to collect weakly more revenue in contrast to the static auction, the experimental literature has supported the dynamic auction for a long period of time because of its price discovery and transparency qualities. This is important since there appears to be a competitive effect, what seems to be a myopic joy of winning, that works in the other direction. That is, many other experiments, starting with Kagel et al. (1987) in an affiliated private value setting, have shown a pervasive bidding above the value in static uniform auctions, whereas this is alleviated in the dynamic auction. This also carries over to CV settings, and Ahlberg (2011) showed, in another multi-unit, CV setting, substantial bidding above value in the static uniform auction. This overbidding affects the profit for the bidders, and often produces negative earnings. Thus,

**Hypothesis 1** The static auction will, at the expense of the bidder profit, deliver the highest revenue of the two formats.

In a common value auction, there is also an adverse selection effect called the winner’s curse (WC). It arises when bidders neglect the information a win will produce, and overbid as a result. The core of the WC is that the announcement of winning the auction leads to a decrease in the estimated value, if not accounted for when bidding. That is, even though the signal in region 2 is ex ante an unbiased estimator of the value, the largest of all bidders’
signals is not. (The \( \max \) function is convex and thus overestimates the value.)

Assume that values are uniformly distributed and the signals are uniformly distributed around the values. Then, if the value, and hence the signal, came from a continuous distribution, the conditional expected value for player \( i \), with realized signal \( s_i \), would be:

\[
E_i(v|s_i > s_{-i}) = s_i - 10 \frac{n - 1}{n + 1} \tag{1}
\]

where \( s_{-i} \) is defined as the realized signals from the other players and \( n \) is the number of players. Thus, in 3-player (6-player) groups, the bidders must scale down their expected value by 5 (7) from the signal to avoid falling prey to the WC. We will use this measure when testing whether bidders account for this adverse selection effect. The hypothesis, partly from Ahlberg (2011) where there was a fairly large amount of WC in the static uniform auction, is that they do not; but, once more, to a lesser degree in the dynamic auction because of its inherent price discovery mechanism. (The survey, by Kagel (1995), of experiments with single-unit auctions also shows the presence of WC, to various degrees, for the inexperienced as well as professionals under a variety of circumstances.)

**Hypothesis 2** The winner’s curse will be present in both auctions, but more so in the static form.

From the above, we had:

**Hypothesis 3** The equilibrium strategy to bid or dropout at the conditional expected value of the first unit should be more likely in the dynamic format due to information revelation, but the problem may be to bid/dropout at the conditional expected value (equation 1) and not at the naive EV \( (s_i \text{ in region 2}) \).

From equation 1, we had that the conditional expected value decreases with the number of bidders. From the Nash equilibrium theory, when there is just one unit for sale, we also have that the bids will decrease with the number of bidders. But, in contrast to this, Kagel et al. (1995) show that bidders fail to respond to the Nash predictions in a single-unit, second-price auction with CV. Ahlberg (2011) also shows this in a multi-unit setting. We believe the experimental literature to have more bearing also in this case; thus, we have that:

**Hypothesis 4** Subjects’ bids will not decrease in response to an increased number of bidders. Thus, instead of halving the supply, increasing the number of bidders, to construct a tighter market, will have the same effect. Hence, the tighter the market, or the more bidders, the larger the revenue.
The phenomenon when bidders reduce demand in favor of a better price is called demand reduction. This happens in a uniform auction since, with a positive probability, bids may determine the price paid on all units. Thus, in every undominated equilibrium, bids other than on the first unit are lower than the expected value. The hypothesis of which of the two formats transmits more demand reduction than the other also hinges on how the dynamic auction behaves relative to the static auction. If we use the theory for interdependent values by Ausubel and Crampton (2002), the dynamic auction should, if there is no collusive behavior, diminish the demand reduction tendencies and thereby give smaller differences between the bids/dropouts. We will measure demand reduction as the spread in players’ bids/dropouts.

**Hypothesis 5** Demand reduction, or bid-spread (dropout-spread), is likely to be in play, but to a lower extent in the dynamic auction.

### 4 Experimental design

Table 1 shows the design of the auction. Each format will have two group sizes, and each group size will have small and large demanders. The demand configuration in 6-player groups is exactly twice that of the smaller group, which, in turn, has two subjects who demand 2 units and one subject who demands 4 units.

<table>
<thead>
<tr>
<th></th>
<th>3-player groups</th>
<th>6-player groups</th>
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<tbody>
<tr>
<td></td>
<td>small</td>
<td>large</td>
</tr>
<tr>
<td>Static auction</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Dynamic auction</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 1
Auction configuration

In the experiment, students from KTH (the Royal Institute of Technology) were used as experimental subjects. They were from different Master of Engineering programs, and the experiment took place in September 2011.

The experiment is between subjects in a fixed matching procedure, i.e. they played against the same competitors and were placed in the same group and had the same demand throughout the session. The subjects were recruited for computer sessions where the given auction mechanism was iterated (unknown to the subjects) 10 times. In each auction, the bidders had the opportunity to

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5 The decision not to communicate the number of rounds was made on the basis that
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In each auction, the bidders had the opportunity to buy as many items as the bidder showed demand for. Before the auction began, the subjects got instructions and, in three trial periods, had the opportunity to become familiar with the interface. The information a bidder got in advance of each round was: the (updated) monetary balance, the own signal (as well as its distribution), own demand, total supply, and how many bidders there were in the auction from the start. Moreover, the subjects were equipped with a starting balance of 50 experimental currency. Bidder instructions are find in Appendix B and C.

In the static version, the subjects simultaneously submitted bids. Then, the software ranked them and made the necessary calculations. Following each auction period, bidders were provided with the true value, the price, the four highest bids along with adherent signals, the number of units won and own profit.

In the dynamic version, the price started with zero for 15 seconds and then increased at a rate of 1 per second. Bidders responded with the quantities demanded at all prices, starting with full demand for all participants at time (price) zero. At any price, bidders were able to drop out on any number of demanded units. When bidder $i$, say, dropped out on 1 or more units, the clock stopped for 5 seconds and increased at a rate of 1 per second thereafter. Any other bidder dropping out during this brief pause was regarded as having the same drop-out price as the first, but later in time. (This five-second delay of time was implemented for every dropout.) Moreover, a dropout was irrevocable. The auction ended when demand equaled supply. If a dropout produced excess supply, the price was rolled back one increment and the bidder (who dropped out) got to buy as many units as were needed to clear supply and demand. The information on the screen during the bidding process was the prevalent price and the number of active bidders and own dropout prices (so far). Then, following each period, the computer screen showed the true value, the price, the four highest dropout prices along with adherent signals, the number of units won and own profit.

The software was developed in Asp.Net framework 2.0 using c# for back-end programming and MsSQL database. The sessions lasted for about 40 to 70 minutes; the open format often took a little longer, but the number of subjects in the session was also a time driver. After each session, all earnings were exchanged into real currency. Each subject earned the same amount in subjects would play differently knowing it was the last round. This could happen if, say, they had lost much money during the first nine rounds, and therefore wanted to gamble a bit.

6 This may seem fast, but it is not; according to the subjects themselves. Moreover, it seems to be the common rate in similar experiments.

7 We also use Ajax for front-end programming to improve the user experience and interact with the database for fast feedback of input/output.
SEK as the monetary balance on his/her screen. Subjects earned, in the mean, SEK 253 (€25) which included a show-up fee of SEK 100 (€10). The minimum earning was SEK 100 (€10), and the maximum earning was SEK 659 (€66).

5 Experimental results

The data description is found in Table 2, which shows, for each format, the number of subjects, how many rounds there were, and the number of unique observations, and the average profit.

<table>
<thead>
<tr>
<th></th>
<th>No. of subjects</th>
<th>No. of rounds</th>
<th>Unique observations</th>
</tr>
</thead>
<tbody>
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<td>Static</td>
<td>64</td>
<td>140</td>
<td>626</td>
</tr>
<tr>
<td>Dynamic</td>
<td>65</td>
<td>149</td>
<td>653</td>
</tr>
</tbody>
</table>

Table 2
Data summary

All comparisons below use statistic tests based on aggregated data over all auction periods, if not stated differently. The non-parametric Wilcoxon (Mann-Whitney) rank sum test has been the main tool, especially between treatments but also within treatments when there is no dependency between the variables. For some comparisons, within a treatment where there is dependency, the non-parametric Wilcoxon signed rank test is employed. We have also tested OLS and panel data (random effects) models with the profit and revenue (price) as the dependent variables. Profit is explained by signal, format, group-size, demand and round, while revenue is explained by value, format, group-size and round. There was only a marginal change in the results presented below and, thus, the conclusions still hold. (The OLS regression on bidder profit can be found in Appendix A.)

When doing the econometric tests, one interesting result was that the region did not matter; only the signal. That is, first it did; the profit was significantly lower (higher) in region 1 (3). But when controlling for the signal, the region became insignificant. Thus, we are using the whole set in the below analysis.

To get a first impression of the data, we plot the bids/dropouts in a scatter diagram. Figure 1 shows the high bid and the second highest bid for the static uniform auction, while figure 2 shows the last and second-to-last dropout prices in the dynamic uniform auction. In the graph for the dynamic format, if a subject has not dropped-out on a unit, the price is registered.

Since 95 percent of all units won, and 80 percent of all clearing prices, in the static auction come from first and second unit bids, the figure comprises
Subjects earned, in the mean, SEK 253 (25) which included a show-up fee of SEK 100 (10). The minimum earning was SEK 100 (10), and the maximum earning was SEK 659 (66).

### 5 Experimental results

The data description is found in Table 2, which shows, for each format, the number of subjects, how many rounds there were, and the number of unique observations, and the average profit.

<table>
<thead>
<tr>
<th>No. of subjects</th>
<th>No. of rounds</th>
<th>Unique observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>64</td>
<td>140</td>
</tr>
<tr>
<td>Dynamic</td>
<td>65</td>
<td>149</td>
</tr>
</tbody>
</table>

Table 2

Data summary

All comparisons below use statistic tests based on aggregated data over all auction periods, if not stated differently. The non-parametric Wilcoxon(-Mann-Whitney) rank sum test has been the main tool, especially between treatments but also within treatments when there is no dependency between the variables. For some comparisons, within a treatment where there is dependency, the non-parametric Wilcoxon signed rank test is employed. We have also tested OLS and panel data (random effects) models with the profit and revenue (price) as the dependent variables. Profit is explained by signal, format, group-size, demand and round, while revenue is explained by value, format, group-size and round. There was only a marginal change in the results presented below and, thus, the conclusions still hold. (The OLS regression on bidder profit can be found in Appendix A.)

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To get a first impression of the data, we plot the bids/dropouts in a scatter diagram. Figure 1 shows the high bid and the second highest bid for the static uniform auction, while figure 2 shows the last and second-to-last dropout prices in the dynamic uniform auction. In the graph for the dynamic format, if a subject has not dropped-out on a unit, the price is registered.

Almost all sales and prices (even though the sales (prices) become slightly overestimated because of the missing 5 (20) percent). But the first impression is nonetheless that there is substantial bidding above the signal for the first unit, with more than half of all first unit bids being greater than the signal. The second unit bid is, by its nature, lower, but it continues to be high for many subjects.

For the dynamic auction, the second to last dropout is when the aggregated demand in the auction shifts from six to five units. The last dropout is when the aggregated demand shifts to four units; that is, where the auction ends and, therefore, also the same as the clearing price in the auction. Thus, the dropouts are auction-specific, unlike the static auction, where the bids are subject-specific.

The first impression in the dynamic auction is that the clearing prices do not seem to be as high as in the static auction; a larger number of the last dropouts are below the signal, although there are quite a few above.

The auctions were not fully effective in that, in theory, the high signal holder(s)
should always win units. If the signal vs. units won relationship is more closely examined, the result becomes the following: Given all subjects with the (weakly) highest signal within each auction round, 86 percent in the static auction won some units, as compared to only 77 percent in the dynamic auction. Instead looking at the (weakly) lowest signal within each auction round, 28 percent in the static auction now won units, as compared to 48 percent in the dynamic auction. Hence, close to twice as many subjects with the weakly lowest signal won units in the dynamic auction as compared to the static auction. Naturally, this affects the profit in the auction. But, it need not be exotic since it is often quite natural for the low signal holder to win units, e.g. when, in 3-player groups, a large demander has the weakly lowest signal. (Then, if the other two small demanders are engaged in demand reduction, the larger demander has a big chance of winning; even though she has the lowest signal.)
5.1 Revenue and profit ranking

The first hypothesis examines the (seller) revenue and the (buyer) profit collected in each auction. Will the auction types give the same revenue in the mean? Or, correspondingly, will they produce an equal profit on average? We start with the profit.

Profit:

First, we look at winning bids only. Then, we have that, on average, the mean of the signal minus the value is almost the same; it only differs at the second decimal, it is 1.22 in the static and 1.29 in the the dynamic auction. Accordingly, in the mean, it was subjects with signals 1.22 (1.29) over the realized value who won the units.

But, even so, each column of table 3 shows the difference between the respective, (realized) value and price, signal and price, and the ratio of the two. Here, it is readily seen that the dynamical auction is superior in raising profit (when we are looking at winning bids only). On average, when looking at the

<table>
<thead>
<tr>
<th></th>
<th>v − p</th>
<th>s − p</th>
<th>s−p/v−p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>1.22</td>
<td>1.99</td>
<td>1.63</td>
</tr>
<tr>
<td>Dynamic</td>
<td>4.54</td>
<td>4.12</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 3
Mean profit, pseudo profit and the ratio of the two.

values minus the prices (first column), it gives almost four times (3.72) as much profit as compared to the static auction. But the pseudo profit, which we define as the signal minus the price, was only 2 (2.07) times as large in the dynamic auction. (The p-values for the first two measures are below 0.01 between auctions.)

Thus, even though the value is on average 1.29 lower than the signal in the dynamic auction, the average profit is higher than the pseudo profit. Whereas, in the static auction, where the value is on average 1.22 lower than the signal, the average profit is lower than the pseudo profit. The two formats go separate ways in this respect which, in turn, makes the dynamic auction perform better for bidders. The last column shows the ratio between the actual and the pseudo profit, where it is seen that the static auction has nearly twice (1.79) as high a ratio as compared to the dynamic auction.

Now we turn to all bids, not just winning ones. Each auction format comprises both 3-player and 6-player groups. And inside each group size, 2/3 of the subjects demanded 2 units (small demanders) and 1/3 demanded 4-units (large demanders). In table 4, we see a highly significant ranking between the group
All players | 3-player groups | 6-player groups
---|---|---
Static auction | 0.67 (8.53) | 5.55 (10.80) | −1.74 (5.82)
Dynamic auction | 2.85 (10.71) | 8.26 (14.53) | −0.28 (5.71)

Table 4
Mean bidder profit in each group size. (Standard deviations inside the brackets)

sizes in each auction format. Regarding the ranking between auction formats, we lose some predicting power when we split them up because of the poor significance between the auctions in 3-player groups (p-value = 0.1354), but in 6-player groups the p-value is 0.0026 and even lower for all players. Continuing

Table 5
Mean bidder profit for small and large demanders, in each group size.

3-player groups | 6-player groups
small | large | small | large
---|---|---|---
Static auction | 5.05 (8.29) | 6.51 (14.50) | -0.95 (4.59) | -3.25 (7.46)
Dynamic auction | 7.72 (12.05) | 9.34 (18.62) | -0.06 (5.30) | -0.75 (6.50)

Thus, overall, we see that the dynamic auction was better at delivering profit to the subjects. When the auctions were split into the two group sizes, the ranking between formats became insignificant in 3-player groups, although the ranking of group sizes inside each format was significantly distinct. But, when the groups were divided into even finer parts, large and small demanders, the ranking between the auctions was partly recovered. But, we must not forget that there was some doubt about the effectiveness of the open auction since quite a large fraction of low signal holders won units, but it could also have a natural explanation (presented in the above subsection). (Effectiveness is not to be confused with efficiency, since all allocations are efficient in a CV auction.)

An interesting property from table 5 is that large demanders in 3-player groups seem to get more profit than do small demanders, but this does not carry over to 6-player groups; in these groups, the large demanders get less profit. Looking more closely at that phenomenon, only data from 6-player groups confirms that large demanders get less profit than small demanders (p = 0.0320). In 3-player groups, even if the data is pooled, there is no statistical difference between small and large demanders.
But, players with larger demand should, especially in 3-player groups, win more units due to their greater demand. Table 6 shows the number of units won, divided into group and demander size for the two auction formats pooled. (There is no significant difference in either group size or demand size between the two formats when comparing units won.) This is only confirmed in 3-

<table>
<thead>
<tr>
<th>3-player groups</th>
<th>6-player groups</th>
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</thead>
<tbody>
<tr>
<td>small large</td>
<td>small large</td>
</tr>
<tr>
<td>Static &amp; Dynamic</td>
<td>1.07 1.80</td>
</tr>
<tr>
<td>Static &amp; Dynamic</td>
<td>0.63 0.77</td>
</tr>
</tbody>
</table>

Table 6
Number of units won, pooled auctions.

player groups, the large demander won 1.8 units in the mean, while the small demander won just over 1 unit.

Hence, large demanders in 3-player groups win 1.68 as many units as small demanders, but they do not earn more profit. If, then, profit per unit won is examined on the pooled set of auctions, table 7 is finally obtained, where the

<table>
<thead>
<tr>
<th>3-player groups</th>
<th>6-player groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>small large</td>
<td>small large</td>
</tr>
<tr>
<td>Static &amp; Dynamic</td>
<td>6.15 4.49</td>
</tr>
<tr>
<td>Static &amp; Dynamic</td>
<td>-0.76 -2.52</td>
</tr>
</tbody>
</table>

Table 7
Mean profit per unit won, pooled auctions.

difference between mean profit per unit won is statistically significant for both group sizes at the 1 percent level.

Thus, in both group sizes, when the auction formats are pooled, the profit per unit won is significantly lower for large demanders, although it was only in 3-player groups that large demanders won significantly more units per auction. As for the total profit per subject, it is only significantly lower in 6-player groups; but it holds for both auction formats. The rationale for this would be aggressiveness; large demanders act as (are) big participants and become price drivers. They outbid small demanders and thus, earn less profit per unit won, but, at least in 3-player groups, they win more units. (More on this in subsection 5.2.)

The fact that only large demanders in 3-player groups won significantly more units is probably explained by the fact that they were the sole large demanders in the auction, and thereby represented half of the aggregated demand. In 6-player groups, there were two large demanders, who together represented half of the demand, but alone only 1/4 of the aggregated demand. Consequently,
since the market is much tighter in the larger group, the large demanders do not have the same price influence as they had in the smaller group sizes.

Ergo, the dynamic auction is the choice for the players. It is naturally better to be in a small group than in a large one, due to the tighter cap to which the bigger groups give rise. Furthermore, group size has more bearing than auction format; the 3-player groups in the static format give a significantly greater profit than 6-player groups in the dynamic auction. When it comes to demand size, there is more ambiguity about the ranking. But solely looking at 6-player groups, subjects with small demands earned less negative profit than large demanders. And, overall, small demanders earned more profit per unit won, but, at least in 3-player groups, they won less units.

Revenue:

Revenue is closely (negatively) affiliated to the profit in CV auctions, i.e. how much money each auction delivers to the auctioneer. The revenue is defined as how much money each round delivers, i.e. the price times four (units). Thus, we now measure between auction rounds, not between subjects.

When using this definition, we do not see any significant differences; neither between formats, nor between group sizes. But when controlling for value, by dividing all prices by the value of the unit, the p-values goes down. Table 8 shows that, overall, the static auction hands over more revenue than does the dynamic auction (p-value = 0.0169). The ranking also seems to extend down to group sizes, as can be seen in the table, but it is only in 6-player groups that the mean values differ significantly (p-value = 0.0090) from each other.

<table>
<thead>
<tr>
<th></th>
<th>Both groups</th>
<th>3-player groups</th>
<th>6-player groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static auction</td>
<td>1.05</td>
<td>0.92</td>
<td>1.18</td>
</tr>
<tr>
<td>Dynamic auction</td>
<td>0.93</td>
<td>0.86</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Table 8
Mean revenue, divided with value.

But, on the horizontal level, i.e. intra comparison, there is no doubt about the natural hypothesis that the more bidders, the more revenue (p-values below 0.0001). Or the tighter the market, the larger (smaller) the revenue (profit).

For the revenue ranking hypothesis, the competitive effect had the largest bearing in the experiment. Participants engaged in overbidding, generally in 6-player groups which, especially in the static auction, led to negative profits in the greater part of all auctions. Table 9 shows the percentage of auctions with negative profits.

Result 1 *The static auction surrenders more revenue to the auctioneer than*
5.2 Winner’s curse

There are two types of overbidding; (i) bids which result in prices above the expected value of the objects, that is the signal ($E(v) = s$), and (ii) bids that result in prices above the conditional expected value ($E(v|s_i > s_{-i}) = s_i - \frac{10}{n+1}$), but below or equal to the expected value (or signal), that is, the winner’s curse (WC) interval. In the experiment, even though the equilibrium bids are unknown in the auctions, there is always a potential risk that a player’s bid becomes the price-setting bid. Thus, bidding above the conditional expected value could be costly.

Starting with the static auction format, 74 percent of the bids where subjects won one or more units were above $E(v|s_i > s_{-i})$. 44 of these were above $E(v)$; hence, taking the difference between the two intervals, we have that 30 percent of the winning bids were in the WC interval. The outcomes in the two group sizes were almost identical in bidding above the signal, category (i), but the outcomes for bids in the WC interval were significantly different. The outcome was 38 percent for 6-player groups as compared to only 25 for the smaller group size, which is seen in table 10.

There is much less overbidding in the dynamic form; as has been noted above as fewer auctions with negative profits. 31 percent of all bids are above $E(v|s_i > s_{-i})$ as compared to 74 in the static form. Bidding above $E(v) = s$ is also much lower, 21 percent as compared to 44 above. Thus, the percentage of bids in the WC interval is just 10 percent compared to 30 above for the static. Moving to group sizes, we note that the proportion between the two group sizes is almost identical to the static auction; but the level in the dynamic auction is just one third of the level of the static auction (table 10).

Bidding above $E(v) = s$ results in a negative profit in the mean, while bidding above $E(v|s_i > s_{-i})$ could, upon winning, give rise to a price that is greater than the estimated worth of the objects and, possibly, create negative profit. It is in this interval that the WC reigns. Looking more closely at winning bids in the WC interval, we get that in the static auction, actually 18 percent fall
prey to the WC; 23 percent in 6-player groups and just 11 percent in 3-player groups. As for the dynamic auction, only 6 percent are accounted for in the WC and 8 (4) percent in 6-player (3-player) groups. Table 10 summarizes bids in this interval, where WCI is an abbreviation for the winner’s curse interval and WC for the actual winner’s curse; that is, the negative profit following from bids in WCI. The dynamic auction has approximately one third of the entries of the static auction; hence, the format produces a much smaller number of bids in the WCI as well as actual WC. In both auctions, the larger group sizes produce 3/2 more bids in the WCI, but approximately 2 times as many WC cases. This is quite natural since the 3/2 more bids in 6-player groups are numerically larger than 3/2 more bids in 3-player groups.

If the group sizes were replaced by small and large demanders in the above table, all entries would be almost exactly the same, barely differing in two slots. This tells us that we have the same results, according to WC, as in table 10 between small and large demanders if groups are pooled. Hence, it is through the large demanders, or 6-player groups, that 2/3 of all WC is encountered.

The dynamic auction behaved as hypothesized, it mitigated much of the WC encountered by subjects in the static auction. It also lessened the pervasive bidding above both expected values. Hence, for bidders, it is the auction of choice, at least inside this model. Thus, the experimental literature has more bearing on behavioral prediction than the theoretical literature. The latter accounts for those players that scale down bids and do not bid above the conditional expected value, but it does not happen in this experiment, especially not in the static auction, with frequent overly aggressive bidding. Theory does not account for bidders who overbid as severely as do subjects in the static auction; it assumes equilibrium play, or play that gives a weakly positive profit in the mean.

**Result 2** There was three times as much bidding in the winner’s curse interval, as well as the experienced winner’s curse, in the static auction estimated relative to the dynamic auction. The static format also had more than two times as much bidding above the standard, naive, expected value.
The result shows that the dynamical auction shifts all bids downward toward more rational bidding even though the bids do not do fully converge to rational bidding. The subjects seem to better understand the laws of demand and supply in the open auction, and also seem to better grasp the idea of a pure common value.

5.3 Equilibrium bidding

Even though the equilibrium strategies are unknown in this game, we know that all weakly undominated equilibria have players who bid the conditional expected value on the first unit. And if we allow bids/drop-outs \( \pm 1 \) of \( E(v|s_i > s_{-i}) \) to also count as correct bids, we have that only six percent are using this strategy in the static auction (fifteen percent meet the naive expected value to bid \( \pm 1 \) of \( s \)); in the dynamic form, the corresponding numbers are sixteen percent (twenty percent meet the naive EV). Thus, there is a relatively larger amount of equilibrium play in the dynamic auction as compared to the static counterpart.

**Result 3** Compared to the static auction, the dynamic auction had more dropouts coinciding with individual rationality, i.e bidding below the expected value; much more for bidding below the conditional expected value.

5.4 Bidding behavior in 3 vs. 6-player groups

The hypothesis of increasing the number of bidders instead of halving the supply to construct a tighter market seemed to be correct, considering the bidding behavior. Subjects’ bids did not decrease in response to the increased number of bidders, contrary to the Nash equilibrium theory (for single unit demand). The null hypotheses that the bids are independent samples from the same distributions cannot be rejected between 3 and 6-player groups in any of the auctions (p-values: 0.5097 and 0.3077.) Moreover, in the revenue-section above, we saw that the more bidders, the higher the revenue.

**Result 4** Increasing the number of bidders instead of halving the supply, to create tighter markets, cannot be rejected as false. Moreover, the tighter the market, the larger the revenue.
5.5 Demand reduction

The last hypothesis concerns demand reduction, which, measured here, translates more into bid/dropout spread; that is, how large is the difference between the first and the second bid, or, for large demanders, the difference between the first and the mean of the three lower bids. This is a crude measure since the above result gave us that roughly just 1/6 of the first unit bid was equilibrium bids, but it gives an indication of demand reduction.

In the comparison between the two auction types, there is a significant difference at the 1 percent level in that there is less bid spread in the dynamic auction; the mean of the spread is 6.43 in the dynamic auction, while it is 8.97 in the static auction. Moreover, it is of no importance if the formats are split into 3- and 6-player groups, or into small and large demanders; the result is approximately the same bid spread, and it is always significant at, at least, the 1 percent level.

This substantial difference partly originates from the fact (described above) that there is considerably more overbidding in the static auction. But the relatively lower spread in the dynamic auction is according to the theory of information dispersion by Wilson (1977); players need not take as much precaution as in the static format, since information about the common value is being updated during the bidding process.

Further elaborating on the bid spread, there is a much greater spread in 6-player groups as compared to 3-player groups. Pooling both formats provides the mean value of 5.44 in 3-player groups and of 8.75 in the larger group sizes (p-value < 0.001). Hence, not only does the bid spread chiefly emanate from the static auction, the larger part comes from the larger group size. But, there is no significant difference between small and large demanders.

Consequently, subjects behave according to the theory of demand reduction. The rationale is that bidders reduce their demand for a more favorable price.

Result 5 There is a widespread demand reduction in both formats, and as predicted by the hypothesis, the spread was somewhat smaller in the dynamic format as compared to the static. Moreover, there was a significant difference between 3- and 6-player groups.
6 Discussion

The results of the experiment in the present paper both contradict and are in line with existing theory. The first hypothesis of the revenue ranking contradicted the existing theoretical literature contention that open formats should deliver more revenue, not less. The present experiment also comes up with a different outcome than Ausubel et al. (2009) who established similar prices for the two formats. But, generally, it is in line with the experimental literature pointing at overly aggressive bidding in the static auction, manifested in that the better part of the auctions often ends up with negative profits for the subjects. And we have seen in this experiment that the dynamic auction cushions much of this bidding above value. Even if it still exists, it is more than halved as compared to the static auction.

Regarding the WC, we distinguished between bidding in the actual WC interval and just bidding above the conditional expected value. We have not seen this before, since experiments often report the latter interval. In the WC interval, subjects experienced three times as much WC, i.e. negative profit, in the static auction as compared to the dynamic format. (Consistently, there were also three times as many bids in the actual interval.) This shows the superiority of the dynamic auction over the static auction in guiding subjects to what the actual common value is in the auction.

The first explanation of the WC should probably be that players in this experiment were inexperienced. They came to the experiment without knowing what to expect. Nevertheless, all players had three dry runs before the experiment, in addition to ten rounds in the experiment. Therefore, subjects at least gained experience along the way. Another explanation is limited liability, meaning that subjects did not have to stand their own losses; they had their starting balance of 50, and had to leave when they went bankrupt. (It only happened 5 times, 3 in the static and 2 in the dynamic auction.) However, Kagel and Levin (1991) and Lind and Plott (1991) provide an experimental verification that limited liability forces did not account for the overly aggressive bidding reported in, at least, their set-up, which is similar to the set-up in the present paper, but with single-unit demand.

There was also much less demand reduction, or bid spread, in the dynamic auction. This is according to theory, but, at the same time, since there was no considerable equilibrium bidding on the first unit, it is hard to evaluate the demand reduction. Still, the variance in the static auction is twice as high as in the dynamic auction, and the standard deviation is also higher in the static auction. This tells us that subjects behave more uniformly in the dynamic form which is, probably, brought forth from the information revelation in the auction.
In IPV settings, some lab and field experiments have showed the superiority of the static uniform auction over the dynamic form, and have also underlined the caution that is warranted in using open formats in multi-unit settings. This does not need to carry over to common value settings. While the static form delivers more revenue than the dynamic form also in the present experiment, it comes at a pretty high cost for the subjects.

The other side of the coin is that, in CV settings, the static auction seems to bring forth an overbidding which is moderated in the dynamic auction. CV auctions are known to produce allocations with negative profits and, in these, the dynamic form could be an excellent guide to price discovery. Why there is this overbidding, especially in the static auction, is hard to tell; it seems as if there is some myopic joy of winning, see Holt and Sherman (1994). The competitive effect takes over the rationality. Consequently, the dynamic form has nice properties for a common value auction, especially for inexperienced bidders.

In both Kagel (1995) and Ausubel et al. (2009), we find strong advocates for the dynamical auction over the static one. The first is a survey of (partly) single-unit, common value experiments establishing that the dynamic form helps alleviate the overbidding in the static format. This also holds for experienced bidders. One of the problems, they argue, is that an increased number of bidders produces no change in bidding in second-price auctions; which it should, according to the robust Nash equilibrium prediction. Subjects encountered the same problem in the present experiment, especially in the static auction; they did not seem to understand that the more bidders in the auction, the bigger the chance of being the price setter and/or bidding above value.

In the paper by Ausubel et al. (2009), the experiment, which is similar to ours, produces equal prices in the two formats, contrary to the present experiment, but, at the same time, they find that the open format is less prone to bidding error and to deliver much higher payoffs. The rationale is that the open environment helps subjects understand complicated settings and thereby reducing errors in finding, if not the optimum, a better outcome than the static auction can contribute to.

The static uniform price auction is the extension of the second-price auction for multi-unit auctions, in the same way as the dynamical uniform auction is the extension of the English auction.


7 Conclusions

In deciding which of the two auction formats of the uniform price auction that were used in our controlled laboratory experiment that is preferred, one has to decide if (i) collecting the most revenue or (ii) avoiding the most negative bidder profit is the most important criterion in the choice process.

If revenue is the most important selection criterion, the static format is the best choice. It generally collects a significantly greater revenue, particularly in a bigger group size. As a result, the profit is greater in the dynamic auction, which holds true (weakly) even if the two formats are split into finer parts; first into large and small group sizes, and then, even finer, into large and small demanders. We also got the corollary that the tighter the market (or the more bidders), the greater the revenue.

On the other hand, if avoiding negative profit is more interesting as the selection criterion, the dynamic auction is better. It only has $1/3$ of the actual WC of the static auction, and less than half of the bidding of the static auction above the conditional expected value. Moreover, almost half of all auctions in the static form terminated with a negative profit for the subjects, as compared to $3/10$ in the dynamic form. Moreover, not only were there more auctions with a negative profit in the static form, the mean of the negative profit was also greater in it.

The weak equilibrium strategy to bid (either one of the two) EVs on the first unit, was also better in the dynamic auction. But both only had a few bids on the (extended) target. No auction format (at either of the two targets) had better than $1/5$ of the bid in the zone.

As for the prevalence of demand reduction, we measured the bid spread and found that the subjects of both formats employed such strategies, but the spread was larger in the static auction. Both the variance and the standard deviation were significantly larger there. But since subjects in neither auction utilized the weak (individual rational) first unit bid strategy, and we know that there was considerable overbidding in the static auction relative to the dynamic auction, it is hard to draw any conclusions.

Auction format is just one feature that determines the outcome of an auction, and our results also illuminate the importance of being in a smaller group size. No matter the format, being in a small group size counts more in terms of bidder profits. But, given the group size, there were different findings for small and large demanders; in 3-player groups, the large demanders earned more profit than small demanders, whereas it was the other way around in 6-player groups.
The bottom line is that, especially for inexperienced players, and for common value settings, the dynamic auction seems to be a better format for price discovery, which mitigates the common overbidding that has been produced in static auction formats.

There is still lack of knowledge in multi-unit settings in general, and in CV settings in particular. The present experiment was carried out with inexperienced subjects, even though they gained experience along the way. But it would be interesting to see how experienced bidders performed in a similar setting. The conclusion from earlier experiments is that they continue to have problems with CV settings.

8 Acknowledgements

We would like to thank Lars Hultkrantz, Jan-Eric Nilsson for valuable comments on the paper. Also, thanks to Jan-Erik Swärdh for important help with econometrics. This study has been conducted within the Centre for Transport Studies (CTS). The author is responsible for any remaining errors.

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There is still lack of knowledge in multi-unit settings in general, and in CV settings in particular. The present experiment was carried out with inexperienced subjects, even though they gained experience along the way. But it would be interesting to see how experienced bidders performed in a similar setting. The conclusion from earlier experiments is that they continue to have problems with CV settings.

Acknowledgements
We would like to thank Lars Hultkrantz, Jan-Eric Nilsson for valuable comments on the paper. Also, thanks to Jan-Erik Svarth for important help with econometrics. This study has been conducted within the Centre for Transport Studies (CTS). The author is responsible for any remaining errors.

References


### Appendix A

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**Notes:**
- **a**: Dependent variable is profit.
- **b**: ****, ** and * denote difference from zero at the one, five and ten percent significance level respectively.

Table 11
Regression on bidder profit
10 Appendix B

**Bidder Instructions for the static, uniform, common value auction**

10.1 Introduction

Hello and welcome. You will participate in an experiment on economic decision-making. The purpose is to study sales by bidding, i.e. through an auction.

You have the opportunity to win money through participation. The show-up fee is SEK 100 (€10), and by learning the rules of the game, you have the opportunity to earn more than that. On the other hand, you could also lose in the process. To ensure that you walk away with at least SEK 100 in your pocket, we give you a starting balance of SEK 50. If you lose this money, you will be excluded from the experiment. Your winnings, and the show-up fee, will be paid in cash after the experiment.

A rule that applies at all times is that all communication between participants is prohibited. If you have any questions, raise your hand and I will come to you and you may ask your question in a whisper. If I believe the question must be answered, I will repeat it to everyone and give the answer.

10.2 Design

**Rounds:** The experiment consists of several rounds. In each round, 4 identical objects, or units, are to be sold through an auction. (How many rounds to actually play will be unknown to you.)

**The commodities:** Each of you has a value associated with owning these units and would like to buy them. We call this the redemption value, which is the same for all units. How many units you want to buy, i.e. your demand, will be seen on your screen, and the number never changes during the game.

**The redemption value:** Before the start of each round, the value of the units is randomly determined through the program. It draws an integer from an array of possible values. The value can never be less than 10, and the maximum is 90. Therefore, the (value) v belongs to the set \{10, 11, \cdots, 89, 90\}. (All values in this range have an equal probability.) However, you will not know what this value is. Instead you will get private information about this value.

**Information:** Even if you do not know the true value, you will receive information that limits the set of possible values. This will be done through a private information signal that is randomly chosen from a range of values.
between the minimum value $v - 10$ and the maximum value $v + 10$. Therefore, (your signal) will belong to the set \{$v - 10, v + 10$\}. (All values in this range have the same probability.) Your signal will also be an integer.

**Example:** Suppose that the true value of the goods is 36, then your signal will be in the set \{36 - 10, 36 + 10\} = \{26, 46\}.

**Opponents:** You can either have two or five opponents. Your group-size will be seen on the screen.

**Bids:** After receiving your information, that is, after you have seen your signal, you should decide what you want to bid for those units that you demand. It is permissible to place equal or different bids for the units.

### 10.3 Instructions

**Buy:** Those who have placed the four highest bids purchase the units. This may be the same person or different people. If there are ties among the (winning) bids, the program will randomly choose the winner(s).

**Price:** The winners pay a price equal to the highest bid that did not win. That is, the highest bid that was rejected. Thus, all winners pay the same price for the units.

**Example:** 4 units are sold. Five people (A, B, C, D, E) have the five highest bids: 25 (A), 23 (B), 19 (C), 15 (D), 12 (E). A, B, C and D purchase the units and everyone pays 12.

**Gain/Loss:** The winners make a profit equal to the difference between the (redemption) value and the price. If the difference is negative, you make a loss.

**Example of profit:** You won one unit, and the price was 42. The value of the unit was 50. You made a profit of 8 ($50 - 42 = 8$).

**Example of a loss:** You won one unit, and the price was 65. The value of the unit was 61. You then made a loss of 4 ($61 - 65 = -4$).

**Note** If you do not have one of the highest bids, nothing happens. The profit is zero.

### 10.4 Practical execution

**Bidding:** You will come to a (web)page where you will see the signal you received, how many units you demand, and how many opponents you have. On basis of this, you place your bids. You bid in the empty boxes and each box represents one unit. Only integer bids from 0 and up to 100 are possible.

**Money:** You will see what your current balance is before every game starts on the screen. The starting balance is 50. If you lose your starting balance, the auction is over for you.

**Lost starting balance:** If someone (or some) lose her starting balance, she/they will no longer participate in the auction. This means that there will be one
between the minimum value \( v - 10 \) and the maximum value \( v + 10 \). Therefore, (your signal) will belong to the set \( \{ v - 10, v + 10 \} \). (All values in this range have the same probability.) Your signal will also be an integer.

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### Instructions

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**Money:** You will see what your current balance is before every game starts on the screen. The starting balance is 50. If you lose your starting balance, the auction is over for you.

**Lost starting balance:** If someone (or some) lose her starting balance, she/they will no longer participate in the auction. This means that there will be one or more person(s) less in the auction. But the auction continues as usual without these people.

**One round:** After you have entered your bid in the fields, press the button "Add bids". When everyone has pressed the button, the bids are ranked. Those who have placed the highest bids purchase units at a price that is determined by the maximum rejected bid.

If there are more winning bids than units for sale, the program randomizes the winners. The balance is recalculated and a new round starts. On the screen, you will see what the value of the units was, the price, the winning bids (as well as the signals from those with winning bids in parenthesis), the units won, and own profits/losses.

**The end:** After a certain number of rounds, the experiment will end. Then press the logout button, and you will come to a page showing what you have earned in the experiment.

### 10.5 Summary

- You will play a certain number of rounds and in each round, 4 identical units are for sale.
- You will play against two or five opponents. You will see the number of opponents on the screen.
- In each round, all players in an auction will have the same redemption value for all demanded units.
- However, each player only gets an informational signal about the true value. Subjects may or may not see the same information as their opponents.
- One can place bids for as many units as one demands, one for each unit. It is permissible to place equal or different bids for the units.
- You start with SEK 50. If you lose this, the experiment is finished for you, and you are excluded from the experiment. But you can also earn more, depending how you and your opponents act.
11 Appendix C

Bidder Instructions for the dynamic, uniform, common value auction

11.1 Introduction

Hello and welcome. You will participate in an experiment on economic decision-making. The purpose is to study sales by bidding, i.e. through an auction.

You have the opportunity to win money through participation. The show-up fee is SEK 100 (€10), and by learning the rules of the game, you have the opportunity to earn more than that. On the other hand, you could also lose in the process. To ensure that you walk away with at least SEK 100 in your pocket, we give you a starting balance of SEK 50. If you lose this money, you will be excluded from the experiment. Your winnings, and your show-up fee, will be paid in cash after the experiment.

A rule that applies at all times is that all communication between participants is prohibited. If you have any questions, raise your hand and I will come to you and you may ask your question in a whisper. If I believe that the question must be answered, I will repeat it to everyone and give the answer.

11.2 Design

Rounds: The experiment consists of several rounds. In each round, 4 identical objects, or units, are to be sold through an auction. (How many rounds to actually play will be unknown to you.)

The commodities: Each of you has a value associated with owning these units and would like to buy them. We call this the redemption value, which is the same for all units. How many units you want to buy, i.e. your demand, will be seen on your screen, and the number never changes during the game.

The redemption value: Before the start of each round, the value of the units is randomly determined through the program. It draws an integer from an array of possible values. The value can never be less than 10, and the maximum is 90. Therefore, the (value) \( v \) belongs to the set \{10, 11, \cdots , 89, 90\}. (All values in this range have an equal probability.) However, you will not know what this value is. Instead, you will get private information about this value.

Information: Even if you do not know the true value, you will receive information that limits the set of possible values. This will be done through a private information signal that is randomly chosen from a range of values
between the minimum value \( v - 10 \) and the maximum value \( v + 10 \). Therefore, (your signal) will belong to the set \( \{v - 10, v + 10\} \). (All values in this range have the same probability.) Your signal will also be an integer.

**Example:** Suppose that the true value of the goods is 36, then your signal will be in the set \( \{36 - 10, 36 + 10\} = \{26, 46\} \).

**Opponents:** You can have either two or five opponents. Your group-size will be seen on the screen.

### 11.3 Instructions

**Auction Procedure:** This auction is not a so-called price auction, i.e. an auction where you place bid(s) for the units. This is a quantity auction; that is, there is a price clock starting from 0 and ticking up to 100, and the players themselves choose when they want to yield their units. Each player starts with demanding all his/her units, but may yield one or more units at any time during the game.

**Auction Time:** The price clock starts at 0 for 15 seconds, then increases at a rate of 1 unit per second. Every time someone gives up one or more units, the price clock stops for 5 seconds. If someone else gives up one or more units during this short break, the same price is registered but later in time. The clock also stops for an additional 5 seconds.

**Auction Stop:** When the number of non-yielded units is equal to the supply of units, the auction automatically ends and all those who still demand units will win them. They will pay the price that cleared the market, for each unit won. That is, the last registered price.

**Example:** 3 players are asking for 2 units each; the supply is 4. Then, as soon as 2 units are yielded, the market clears, since demand is then equal to supply. The price that everyone pays for each of their units won is equal to the price that cleared the auction; that is, the price that prevailed when the second unit was yielded.

**Excess Supply:** If a bidder yields more than one unit, and thus gives rise to an oversupply, the clock will be rolled back one increment, and the player who made this happen may purchase the same number of units to clear the auction. All players who have won units may then also buy at the new price.

**Example:** Suppose that the price clock is at 49, and 5 demanded units remain in the auction. If a player then yields 2 units when the price clock turns to 50, the aggregated demand drops to only 3 units, while the supply is 4. Then, the player who yielded 2 units may only yield 1 unit, but the price is rolled back to 49. This price applies to everyone who won units.
11.4 Practical execution

**Auction start:** You will come to a (web)page where you will see how many units you demand, how many opponents you have, and your balance. When the auction starts you will also get your signal. From then on, you can yield units. You also have 15 seconds to think before the price clock starts.

**Money:** On the screen you will see your updated balance after each round. The starting balance is fifty. If you lose your starting balance, the auction is finished for you.

**Lost starting balance:** If someone (or some) loses her starting balance, she/they will not participate in the auction any more. This means that there will be one (or more) person(s) less in the auction. But the auction continues as usual without them.

**One round:** After each round, the balance is re-calculated and a new round starts. On the screen you will see what the true value of the units was in the round before, the price of the units, what price the price-clock registered for the four most recent (highest) yielded units (as well as the signal these players had in parenthesis), units won, and the profit/loss in the round.

**Gain/Loss:** The winners make a profit equal to the difference between the (redemption) value and the price. If the difference is negative, you will make a loss.

- **Example of profit:** You won one unit and the price was 42. The value of the unit was 50. You made a profit of 8 (50 − 42 = 8).
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**Note** If you yield all your units, nothing happens. The profit is zero.

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- You will play a certain number of rounds, and in each round 4 identical units are for sale.
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- When you see the signal, the auction has started. Then you demand all your units, but can yield a unit at any time. You can yield one, or more units, depending on what you think is the best. After fifteen seconds, the price clock starts.
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• When you see the signal, the auction has started. Then you demand all your units, but can yield a unit at any time. You can yield one, or more units, depending on what you think is the best. After fifteen seconds, the price clock starts.

34• When demand equals supply, i.e. when there are only four units left, the auction ends automatically.
• You start with SEK 50. If you lose this, you are bankrupt, and you are excluded from the auction. But you can also earn more, depending how you and your opponents act.
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