DTA simulation convergence with reduced number of iterations

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1 Research question and brief overview of the state of the art

An iterated (day-to-day) dynamic traffic assignment (DTA) is considered. In each iteration, a certain fraction of the total travel demand adjusts its travel pattern before the entire demand is loaded onto the (congested) network. It is assumed that the demand is composed of at least two commodities that exhibit non-identical travel behaviors. The demand changes from one DTA iteration to the next can then be decomposed into two components: (i) *demand exchanges* between commodities, which do not affect the total (aggregate over all commodities) demand pattern, and (ii) *demand shifts*, which do affect the total demand pattern. Given that there are many (day-to-day) sequences of demand and congestion patterns that lead to the same stationary assignment, the approach taken here is to minimize congestion variability along this path by favoring, to the extent possible, *demand exchanges* over *demand shifts*. The underlying intuition is that the resulting reduction in congestion variability and consequently better predictability of travel times leads to a faster and/or more stable convergence of the assignment.

The selection of replanning rates in general-purpose DTA simulation packages has, due to the limited tractability of such systems, received limited attention in the literature. In deterministic settings, flow averaging schemes according to (variants of) the method of successive averages are often considered (Liu et al., 2007). In stochastic process assignment, constant replanning rates may be preferred because these can lead to unique stationary process distributions (Watling and Hazelton, 2003). The possibly interesting twist of the approach described here is that it (i) leaves the choice of a global replanning rate (being identical for all commodities) to any available algorithm or expert and then (ii) accelerates the convergence of the simulation process by deriving commodity-specific replanning rates. Lu and Mahmassani (2007) and later Lu et al. (2009) pursue the same line of thinking but make more specific modeling assumptions (a route swapping assignment that aims at attaining a deterministic user equilibrium) than what is assumed in the subsequently described method.

2 Method

Consider a single not yet converged iteration of the assignment. The demand is for now assumed to consist of a single commodity. Assuming discrete within-day time, let i denote a (link, time bin) tuple, subsequently called a "slot". Let x_i be the amount of demand currently using slot i, and let x_i^* be the amount of demand that would want to use slot i if a 100% replanning rate was allowed for. Similarly, let $\Delta u = u^* - u$ be the total expected change in travel utility given a 100% replanning rate.

Assuming a deterministic continuum model, convergence of the assignment process means that $\Delta u = 0$ and $x_i = x_i^*$ is attained for all slots i. In non-converged conditions, the replanning rate λ defines what fraction of demand is shifted from the current pattern $\{x_i\}$ to the desired pattern $\{x_i^*\}$. Setting λ has to balance two objectives: (i) staying near the current demand pattern $\{x_i\}$ in order to avoid oscillations and (ii) moving towards $\{x_i^*\}$ resp. u^* in order to make progress towards a solution. Letting $\Delta x_i = x_i^* - x_i$, the following objective function represents this situation:

$$Q(\lambda) \quad = \quad \sum_i (\lambda \Delta x_i)^2 + \beta(1-\lambda)\Delta u + \delta\lambda^2 \tag{1}$$

Noting that $\lambda\Delta x_i$ is the expected flow change on slot i given replanning rate λ, the first term is a sum of squared link flow changes that penalizes large replanning rates that make large steps away from the current point. With $(1-\lambda)$ being the fraction of demand that does *not* replan, the second term penalizes the loss in expected total utility gain in the presence of *non*-replanning. In combination, these first two terms model the aforementioned preference for *demand exchanges* over *demand shifts*. A (brief) discussion of the non-negative δ parameter is postponed to the analysis section. Evaluating the optimality conditions for minimizing (1), one obtains that choosing the weight

$$\beta \quad = \quad 2\bar{\lambda} \cdot \frac{\sum_i \Delta x_i^2 + \delta}{\Delta u} \tag{2}$$

ensures that the minimizer of (1) equals some desired value $\bar{\lambda}$, independently of the concrete value of δ. This circular problem statement alone is not of practical use. Its sole purpose is to formalize the reasoning behind the choice of a particular step size parameter $\bar{\lambda}$. This formalism can then be carried over to the subsequently discussed multi-commodity setting.

To set step sizes in the case of multiple commodities, the individual terms in the objective function (1) are expressed in terms of an inhomogeneous demand, with the nth commodity replanning at an individual rate $\lambda_n \in [0, 1]$. Specifically, (i) slot i's expected multi-commodity flow change becomes $\sum_n \lambda_n \Delta x_{ni}$ with Δx_{ni} the expected flow change of commodity n i slot i, (ii) the multi-commodity utility improvement becomes $\sum_n (1 - \lambda_n)\Delta u_n$ with Δu_n being the expected utility improvement of commodity n, and (iii) the (now utility-weighted) average replanning rate becomes $\sum_n \frac{\Delta u_n}{\Delta u}\lambda_n$. Combining these terms according to the same reasoning that led to (1) yields

$$Q(\{\lambda_n\}) \quad = \quad \sum_i \left(\sum_n \lambda_n \Delta x_{ni} \right)^2 + \beta \sum_n (1 - \lambda_n)\Delta u_n + \delta \frac{1}{\Delta u^2} \left(\sum_n \lambda_n \Delta u_n \right)^2 . \tag{3}$$

This objective function can now be minimized with respect to individual replanning rates per commodity, subject to the constraints $\lambda_n \in [0, 1]$ or $\lambda_n \in \{0, 1\}$ for all n. The weight β used here is still given by (2) and hence needs to be parameterized by the single-commodity replanning rate $\bar{\lambda}$.

3 Analysis and results

Letting $\{\lambda_n^*\}$ be the real-valued minimizers of (3) for a given $(\bar{\lambda}, \delta)$-parameterization, the convenient quadratic form of (3) allows to rather straightforwardly derive the following properties, with proofs being omitted due to space restrictions: (i) As $\delta^{-1} \sum_i \Delta x_i^2 \to 0$, a minimization of (3) with the weighting (2) yields uniform replanning rates $\lambda_n = \lambda^*$ for all n. (ii) Making $\bar{\lambda}$ sufficiently small reduces the (sum-of-squares) change of slot flows from one iteration to the next to an arbitrarily small amount: $\sum_i \left(\sum_n \lambda_n^* \Delta x_i\right)^2 \leq 2\bar{\lambda} \sum_i \Delta x_i^2$. (iii) Choosing a positive $\bar{\lambda}$ ensures that some utility improvement is attained: $\sum_n \lambda_n^* \Delta u_n \geq \frac{\bar{\lambda}}{2} \Delta u$.

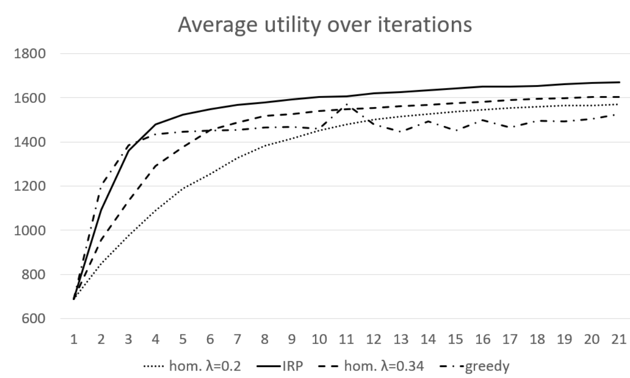

Figure 1: Preliminary results for Stockholm

A preliminary case study with a Stockholm simulation model is considered. A travel demand consisting of 7300 person commodities is assigned to a network of 16'384 links in a a 24-hour simulation with a congested morning and evening peak. Both route and departure time choice are iteratively equilibrated. In every iteration of the simulation, the heuristic of Merz and Freisleben (2002) is deployed to obtain binary replanning indicators for every single commodity such that (3) is approximately minimized.

Figure 1 presents first results. The dotted line illustrates the pace at which the average utility of the traveler population increases over simulation iterations given that a uniform replanning rate of $\bar{\lambda} = 0.2$ is used. The solid line demonstrates the acceleration effect of the proposed recipe, also using $\bar{\lambda} = 0.2$, as well as $\delta = 0$. A discussion of the dashed resp. dot-dashed line is omitted due to space restrictions. Experimentation with this case study is ongoing; comprehensive analysis results can credibly be expected by the time of the conference.

References

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