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The marginal cost of track reinvestments in the Swedish railway network: Using data to compare methods

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Keywords: railways; reinvestment; renewal; survival model; corner solution model; two-part model; marginal cost

JEL Codes: C41; H54; L92; R48

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The marginal cost of track reinvestments in the Swedish railway network:

Using data to compare methods

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Abstract

In this study, we analyze the difference between survival and corner solution models in estimating the marginal cost of reinvestments. Both approaches describe the reinvestment process in rather intuitively similar ways but have several methodological distinctions. We use Swedish railway data on track segment and section levels over the period 1999-2016 and focus on reinvestments in track superstructure. Results suggest the marginal costs from survival and corner solution models are SEK 0.0041 and SEK 0.0103, respectively. The conclusion is that the corner solution model is more appropriate, as this method consider the impact traffic has on the risk of reinvestment as well as on the size of the reinvestment cost. The survival approach does not consider the latter, which is problematic when we have systematic variations in costs due to traffic and infrastructure characteristics.

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Sammanfattning

Denna studie analyserar skillnader mellan två ekonometriska metoder – en överlevnadsmodell och en modell för hörnlösningar – och deras implikationer för skattade marginalkostnader för reinvesteringar i järnvägsspår. Båda ansatserna beskriver reinvesteringsprocessen på delvis liknande sätt, samtidigt som det finns tydliga metodologiska skillnader. Överlevnadsmodellen anger att risken för (och eller tiden till) en reinvestering är en funktion av trafik och olika infrastrukturegenskaper. Modellen för hörnlösningar delar in reinvesteringsprocessen i två delar; dels i ett beslut att genomföra en reinvestering, dels hur stor reinvesteringen ska vara.

Vi använder data över det svenska järnvägsnätet under perioden 1999–2016 och fokuserar på reinvesteringskostnader i järnvägsspår (mer specifikt banöverbyggnad där vi har exkluderat spårväxlar). Resultaten från överlevnadsanalysen visar att marginalkostnaden är SEK 0.0041 per bruttotonkilometer, medan modellen för hörnlösningar ger en marginalkostnad på SEK 0.0103. Skillnaden i estimaten pekar på vikten av att undersöka vilken metod som är lämpligast i denna typ av kostnadsskattning.

Modellen för hörnlösningar är att föredra då metoden kan användas för att få fram trafikens effekt på både sannolikheten för en reinvestering och storleken på reinvesteringskostnaden. Överlevnadsmodellen tar inte hänsyn till den senare effekten, vilket är ett problem när det finns systematiska variationer i kostnader som kan förklaras av trafik och olika infrastrukturegenskaper. Att ta hänsyn till hur trafiken påverkar storleken på reinvesteringskostnaden är särskilt viktigt i vår studie då vi i datamaterialet kan observera reinvesteringar på delar av spårindividerna. Slutsatsen är att vi föredrar modellen för hörnlösningar i denna kontext och rekommenderar marginalkostnaden SEK 0.0103 per bruttotonkilometer för reinvesteringar i järnvägsspår.

1. Introduction

An efficient use of railway infrastructure inter alia presumes that the price for using the system is based on the costs inflicted on the Infrastructure Manager (IM); usage is measured by traffic volume (gross ton kilometers or train kilometers). This rationalizes one of the components of the SERA Directive (2012/34/EU).

Starting with Johansson and Nilsson (2004), there is now a series of econometric analyses of how railway traffic affects the costs for day-to-day maintenance (Andersson (2008), Link et al. (2008), Wheat and Smith (2008), Wheat et al. (2009), Odolinski and Nilsson (2017)). With a double-log specification, the coefficient estimate of the traffic volume variable indicates the cost elasticity, which is then multiplied with predicted average costs to calculate the estimate of marginal costs. This literature has generated estimates of marginal costs that are relatively stable, both within countries as data accumulates over time and in the elasticities between countries.

Analysis of reinvestment activities is less mature. It involves the impact of traffic on the renewal of tracks – rail, ballast, sleeper, switches etc. – and of fixed equipment for providing electricity to trains as well as signaling and telecommunication equipment. The frequency of reinvestment activities may depend on the intensity of traffic, but in a more indirect way than maintenance. Traffic variations may thus change the timing of the activity. Since this also affects the present value of future spending on renewal, it is part of the marginal renewal cost; cf. Nilsson et al. (2015) for a detailed account of this process for road surface renewal.

Andersson et al. (2012) and Andersson et al. (2016) provide two estimates of the marginal cost for track renewal. These papers however use different modelling approaches, a corner solution model and a parametric survival model.[1] The purpose of this paper is to consider the methodological qualities of these approaches and – using information about infrastructure renewal in Sweden from 1999 to 2016 – to estimate the marginal costs using the respective models. This makes it possible to recommend the best method or indeed to establish that either approach can be used

The rest of the paper is organized as follows. Section 2 presents corner solution and survival models. Data and descriptive statistics are described in section 3. Results and its discussions are presented in section 4. The last section concludes the paper.

2. The models

As costs for day-to-day maintenance increase with age and use, and as the risk for technical failures affecting train traffic also increases, the plant must at some point of time be replaced. In view of the robustness of tracks and structures, their renewal is a rare feature often appearing with at least 25 years in-between.

The corner solution model used by Andersson et al. (2012) – described in more detail in section 2.1 – handles these decisions as an outcome of deliberate optimization. Since most parts – most track sections – of the network are *not* renovated most years, an inventory of renewal activities has zero values for the dependent variable, the reinvestment cost. These are true zeros since the infrastructure manager has decided whether to make a reinvestment (observation is 1) or not (0) on each section,

[1] Yet another approach is used by Odolinski and Wheat (2018), who apply a vector autoregressive model to rail infrastructure renewals and maintenance. As the present paper focuses on renewals only, a thorough comparison with the vector autoregressive model is beyond the scope of this paper.

meaning that the use of standard econometric techniques with zero dependent values is not suitable. Instead, a corner solution model (Wooldridge (2002)), where the reinvestment variable takes zero values with a positive probability, is used. On parts of the tracks that are renewed, costs – a positive and continuous variable – are observed and can be analyzed, given the first decision step.

There are three approaches that can be used within the corner solution framework, *viz.* the two-part, the Tobit and the Heckit models. Previous studies in the railway context (Andersson et al. (2012); Yarmukhamedov et al. (2016)), which use the same type of data but with different observation periods, provide indications in favor of the two-part model. Our formal testing also provides support for this choice. Results are suppressed for expositional convenience.

Survival analysis – see section 2.2 for a more detailed description – is an alternative to the corner solution model and is used by Andersson et al. (2016). This approach considers the risk that an asset will "die" (be reinvested), which also refers to an underlying optimization process. The idea is that the survival of assets has generic qualities that can be assessed by using some common idea of the risk of "death". In the present context, a Weibull model is used for capturing these features, meaning that we assume the life of the railway tracks follows a Weibull distribution. This distribution is flexible with a scale and shape parameter, where the latter allows for either increasing, decreasing or constant hazard rates (risk of "death"). In the case for railways it is increasing, i.e. the risk that a track's lifetime will end in the current year is increasing with time and/or with cumulative use.

2.1 The corner solution model

The two-part model explains the reinvestment decision in the first part and the size of reinvestment in the second. The reinvestment decision for track section *i* is specified as a probit model:[2]

$$z_i^* = \alpha_1 + x_{1i}'\beta_1 + u_{1i} \qquad \text{Eq.1}$$

where $I_i = 1$ if $z_i^* > 0$, $I_i = 0$ otherwise, $u_{1i} \sim N(0,1)$. z^* is a latent variable, which describes the decision whether to renew or not, I takes the value 1 (or zero) when a decision is taken to implement a reinvestment (or not), x_{1i}' is a vector of explanatory variables that includes traffic, railway network characteristics, geographical location, and period dummies, α_1 is a constant term and u_{1i} is the error term.

The size of the reinvestment y is specified as a truncated regression model in the second part:

$$y_i|(I_i = 1) = \alpha_2 + x_{2i}'\beta_2 + u_{2i} \qquad \text{Eq.2}$$

where α_2 is a constant term and the expected value of the error term u_2 is zero for positive values of reinvestment costs and the error term is not necessarily normally distributed.

[2] Note that the observations in survival model are on track segment level (*j*), whereas in corner solution model they are on track section level (*i*). The distinction between observation levels are discussed in the data section.

The two-part model allows the process for the decision to undertake renewal activities to be different from the process for the decision on the size of the reinvestment. Thus, the explanatory variables used in the renewal decision process can be different from the ones used in determining the size of the reinvestment. Even if these predictors are the same, their coefficient estimates in terms of sign, magnitude and significance level can be different.

The equation for estimating the marginal cost for track section *i* with respect to traffic volume *k* is:

$$MC_{ik} = \gamma_{ik}\widehat{AC}_i \qquad \text{Eq.3}$$

where γ_{ik} is the cost elasticity:

$$\gamma_{ik} = \frac{\partial E[y]}{\partial x_k} \times \frac{x_k}{E[y]} = \beta_{2k} + \beta_{1k}\lambda(x_1'\hat{\beta}_1) \qquad \text{Eq. 4}$$

β_{1k} and β_{2k} are coefficient estimates for the traffic volume variable from the first and the second parts, respectively; $\lambda(x_1'\hat{\beta}_1)$ is the inverse Mills ratio, $\lambda(x_1'\hat{\beta}_1) = \frac{\phi(x_1'\hat{\beta}_1)}{\Phi(x_1'\hat{\beta}_1)}$, where $\phi(.)$ and $\Phi(.)$ are the probability density and cumulative distribution functions derived from the probit model (Eq. 1); $\widehat{AC}$ is the predicted average reinvestment cost per gross ton-km obtained from the truncated regression model (Eq.2). Specifically, predicted costs are $\mathrm{E}[y] = \Phi(x_1'\hat{\beta}_1)\exp(x_{2i}'\hat{\beta}_2)\exp(\frac{1}{2}\hat{\sigma}^2)$, assuming normally distributed and homoscedastic error terms (Dow and Norton (2003))[3], which are then divided by gross ton-km to get average maintenance cost $\widehat{AC}$.

To take traffic share per track section into account in the marginal cost estimation, we compute a weighted marginal cost:

$$MC^W = \sum_{it} MC_{it} \cdot \frac{GTKM_{it}}{(\sum_{it} GTKM_{it})} \qquad \text{Eq. 5}$$

2.2 The parametric survival model

The survival analysis considers the time to the occurrence of asset renewal. From the point of time when a section of the network is opened for traffic, it is used until the quality reaches a point where it should be renewed. This is the track's lifetime, which is also referred as survival time or time to failure.

Parametric survival models analyze the dependence of survival time *t* at track segment *j* (Cleves et al. (2004)):

[3] Indeed, our error terms are normally distributed according to a Quantile-normal plot (the error terms lie along the reference line for the inverse normal distribution), and a plot of the residuals against the predicted value from the truncated regression does not indicate heteroskedasticity.

$$ln(t_j) = \beta_0 + x_j'\beta_x + \varepsilon_j \qquad \text{Eq.6}$$

where x_j' is the same vector of explanatory variables that are used in the corner solution model, i.e. traffic, railway network characteristics etc.; $\beta_0$and β_x are parameters to be estimated, and $\varepsilon_j$is an error term. Alternatively, Eq. 6 can be expressed as a hazard function, which in proportional hazard (PH) metric form is:

$$h(t) = h_0(t)exp(x_j\beta_x) \qquad \text{Eq. 7}$$

$h_0(t)$ is a baseline hazard, i.e. the change in the risk of reinvestment over time when covariates are zero. Eq. 6 and Eq. 7 are equivalent representations of the same parametric survival model, while the latter expression is more common in recent studies.

There are several parametric models in survival analysis, which differ in terms of the assumption on the distribution of survival time (in Eq. 6) or the functional form for the baseline hazard (in Eq. 7). In the road and railway context, baseline hazard is assumed to follow a Weibull distribution (Link and Nilsson (2005); Nilsson et al. (2015); Andersson et al. (2016)), which is expressed:

$$h_0(t) = pt^{p-1}exp(\beta_0) \qquad \text{Eq. 8}$$

where *p* is a shape parameter $(p > 0)$ and $exp(\beta_0)$ is the scale parameter. Then, the Weibull regression model in PH metric is:

$$h(t_j) = pt^{p-1}\exp(\beta_0 + x_j\beta_x) \qquad \text{Eq. 9}$$

where β_x is the change in the risk of reinvestment due to a one-unit change in x_j. To estimate the marginal cost of track reinvestment with respect to traffic, we are however interested in the deterioration elasticity, that is, the change in the track's life time (rather than risk of reinvestment) due to a traffic increase (Lindberg (2002); Haraldsson (2007)). Thus, we use a slightly different model called accelerated failure time (AFT) model:

$$\tau_j = \exp(-x_j\beta_x)\, t_j \qquad \text{Eq. 10}$$

where $\exp(-x_j\beta_x)$ is the acceleration parameter, and $\tau_j \sim \text{Weibull}\ (\beta_0, p)$ in the Weibull regression model. In this specification, β_x measures the change in the track's survival time due to an increase in x_j. Note that, in the case of the Weibull distribution, the PH model results are easily reparametrized as an AFT counterpart by:

$$\beta_{AFT} = -\frac{\beta_{PH}}{p} \qquad \text{Eq. 11}$$

The parameter estimates in Eq. 9 are obtained using maximum likelihood.

The equation for estimating the marginal cost in the survival analysis is (Haraldsson (2007); Andersson et al. (2016)):

$$MC_j = -\beta_{GT} \frac{c}{\bar{q}_1 \mu_j} \frac{r}{[1-exp(-r\bar{T})]} \int_0^{\infty} exp\left(-r\omega - \varphi_j \omega^p\right) d\omega \qquad \text{Eq. 12}$$

where β_{GT} is a traffic elasticity estimate, i.e. the parameter estimate for the natural logarithm of tonnage density (the gross ton-km per route-km); c is average track renewal cost per renewed track segment length, i.e. the sum of all renewal costs over the observation period divided by the total length of renewed track segment length over the same observation period; $\bar{q}_1$ is a constant average annual traffic volume of the first renewal interval; μ is the expected value of the renewal interval. Specifically, $\mu = E(T) = \frac{\Gamma(1+1/p)}{\lambda^{1/p}}$, where Γ is the Gamma function, and T is time to renewal. r is the social discount rate (4 per cent); $\bar{T}$ is the constant renewal interval; ω is the remaining life time of a track segment, $\omega = T - \tilde{t}$, where $\tilde{t}$ is the renewal time; φ is the scale parameter, $\varphi = \exp(\beta_0)$. As in the corner solution model, we use eq. 5 to calculate the weighted marginal cost.

3. Data

The Swedish Transport Administration (Trafikverket) is responsible for the maintenance and renewal of 85% of the railway network in Sweden. The rest of the network consists of privately owned sections, heritage railways, and siding and track sections that are closed for traffic. Trafikverkets railway system comprises 260 track sections including 24 station sections, where annual spending on reinvestment is about SEK 2.4 billion (2016).

Importantly, each track section comprises several track segments. The data base that provides network characteristics are on track segment level. This includes traffic, track segment length, sleeper type, number of joints, rail weight, quality class (linked to maximum line speed allowed) and regional indicators. Cost data is, however, only available on the more aggregate track section level. Costs are reported for track superstructure (rail, switches), track substructure (bar, intersection, culvert, bridge, tunnel), marshalling yards (lights, train warming etc.), electrical wiring (overhead line, traction power network, transformer etc.), signaling (positioning system, balise, train control system etc.), telecommunications (detector, radio, tele transmission system etc.) and other equipment (property, canalization, drainage and pump systems etc.).

Data on track sections with sparse traffic, industrial tracks, marshalling yards and privately-owned sections are excluded from the analysis due to the insufficient, lacking and irrelevant (marshalling yards: cost structure compared to other track sections can be different at large) data.

Since the survival analysis does not explicitly account for costs, it is possible to use the more disaggregate information, i.e. the track segment level information, generating more observations.

Focus is on track superstructure since information on the life time and renewal date of other constructions (track substructure, marshalling yard, electrical wiring, signal and telecommunication equipment) is not available. Moreover, we exclude switches since the timing of their renewals are different from other parts of the track superstructure.

In the corner solution model, and since the cost data is not available on track segment level, the analysis focuses on track sections. This means that the track segment network characteristics are aggregated to the track section level, i.e. the sum of the segment length and number of joints, as well as the average of quality class and rail weight per track section are computed. To make the results from survival and corner solution models comparable, we focus on track superstructure renewals (excluding switches) in the corner solution model as well, i.e. other cost categories are excluded.

Our data consist of 2 270 track segments or 163 track sections observed over the period 1999-2016. This amounts to 44 349 observations for the survival model (however, this panel data structure is converted to cross-sectional data, i.e. 2 270 observations are used in the model estimation; see next section) while 2 880 observations are available for estimating the corner solution model.

3.1 Descriptive statistics

Using track segment level data for estimation of the parametric survival model, 71 per cent of the segments are censored and 29 per cent are uncensored (Table 1). Censored observations refers to track segments that have not been treated (renewed) sometime between 1999 and 2016, while uncensored observations are sections that have been renewed during this period.[4] In the estimation of the survival model, we only use information from the year prior to the renewal (uncensored segments) and from the last year of the observation period for the segments that have not been renewed (censored segments).[5] This implies that the panel data structure is converted to cross-sectional data (presented in Table 1), where each segment is observed once, yet in different years depending on when, and if, it is renewed.

A closer look at the segments with respect to renewal status (Table 1, column 2 and 3) confirms intuition in that the mean rail age prior to renewal is higher for renewed track segments (29.1 years) than the non-renewed ones (27.9 years). There is a higher tonnage density for the uncensored subsample (7.65 million gross tons) compared to the censored one (5.50 million gross), which implies that heavily loaded segments are given priority. The share of segments with concrete sleepers is higher for censored observations, while the mean rail weight is higher for uncensored observations (i.e. weight prior to renewal), which also belong to a station section to a higher degree compared to the censored observations. The average track quality is higher on the uncensored segments which is clear from the quality class; a low number implies high quality, i.e. high maximum line speed. There are also regional differences between the two subsamples, i.e. the share of renewed segments is higher in the northern region, while there are more non-renewed segments in the eastern region.

As mentioned previously, the track segment data are aggregated to track section level for the corner solution model. However, note that cross-section data for segments are used in the survival model,

[4] It should be noted that only a part of the segment may be renewed, making the age of the track segment an average age.

[5] However, as described previously, for traffic we use the average tonnage density during the observation period prior to the renewal (for uncensored segments) or the average tonnage density for the entire observation period (for censored segments).

while a panel of segments are aggregated to sections in the corner solution model. The average track reinvestment cost per track section and year is SEK 3.66 million (SD 21.71). This cost data is used in both approaches.

Table 1. Descriptive statistics for track segments and sections.

	Track segment (cross-section data), censored N=1613		Track segment (cross-section data), uncensored N=679		Track sections (panel data) N=2880	
	Mean	SD	Mean	SD	Mean	SD
Rail age, years[a]	27.90	9.48	29.13	11.92	20.70	9.25
Ton density* (ton-km per track-km)[a]	5.50	4.55	7.65	5.82	4.90	4.11
Track segment or section length, km	3.71	4.10	3.42	3.64	55.45	42.14
Number of joints	8.97	11.65	9.34	11.11	179.57	140.14
Concrete sleepers (dummy)[a]	0.75	0.43	0.48	0.50	0.67	0.47
Belongs to station section (dummy)[a]	0.04	0.19	0.06	0.24	0.12	0.32
Average quality class, 1 to 6[a]	1.67	1.30	2.13	1.22	3.09	1.10
Average rail weight, kg[a]	52.92	5.56	50.23	4.18	51.50	4.71
West region	0.19	0.39	0.21	0.41	0.16	0.37
North region[a]	0.13	0.33	0.19	0.40	0.13	0.34
Central region	0.23	0.42	0.20	0.40	0.20	0.40
South region	0.20	0.40	0.23	0.42	0.24	0.43
East region[a]	0.26	0.44	0.17	0.37	0.26	0.44

* In millions. Note: the t-test is conducted to investigate the significant difference of mean values from censored and uncensored subsamples (columns 2 and 3), where superscript [a] indicates significant difference at 1% level. Only the variables with significantly different means are commented.

It should be noted that the traffic variable (tonnage density) for censored observations is an average segment tonnage density over the observation period, while for uncensored observations it is an average segment tonnage density from the first observation period up to the preceding period of the reinvestment occasion. For instance, if reinvestment takes place in period 7, we take the average of tonnage density over the 6 periods prior to the reinvestment. In this way, the risk of ignoring traffic fluctuations in previous periods is reduced.

Reinvestment activities are rare events. Therefore, it is quite seldom we observe the second reinvestment within the observation period in our analysis. We observe more than one reinvestment in 5% of the cases, which might be related to the residual/remaining reinvestments after a major reinvestment, or that only a part of the segment was reinvested in the first instant. Otherwise, modelling multiple (recurrent) events with Weibull distribution might be an alternative to the current specification (single event or time to the first event analysis).

4. Results

The corner solution model and the life cycle model results are presented in sections 4.1 and 4.2, respectively. In both approaches, models are estimated both with and without variables describing the technical quality of the tracks (rail weight, quality class, number of joints, and whether concrete sleepers are used or not). The reason is that previous studies did not include all these variables, and their impact on the results can now be tested. We start with a Translog function (see Christensen et al. (1971)) in the survival analysis and test down. For comparison, the same Translog function is used in the probit model, which is the selection equation in the two-part model (the preferred Translog function in the probit model generates a similar estimate for the impact of traffic on the probability of renewal).[6] The marginal costs and differences in results from both models are discussed in section 4.3.

Variables with second order terms have been divided by their sample median prior to taking logs. In that way, the first order coefficients can be interpreted as estimates at the sample median. All estimations are carried out using Stata 12 (StataCorp 2011).

4.1 Corner solution model results

The estimation results from the corner solution model are presented in Tables 2 and 3. In Model 1, the quality variables are excluded, while these variables are included in Model 2. Importantly, the inclusion of quality variables has a large impact on the estimates for traffic. Considering that, for example, the correlation coefficient between the logarithm of quality class and the logarithm of gross tons is -0.5357, omitting the former variable implies that the estimate for the latter will be biased (and vice versa). We therefore focus on the Model 2 results.

The results indicate that an increase in track section length leads to higher probability of renewal (yet, not statistically significant in Model 2) and higher reinvestment costs for a track section. Rail weight has a positive first order coefficient in the selection equation, however, there is a significant second order effect that is negative. Plotting these estimates against rail weight shows that the probability of a renewal decreases with rail weight, which is expected. Joints has a significant effect in both the selection and outcome equations, indicating that more joints result in a higher probability of renewal and that the cost of the renewal increases with the number of joints.

The main interest in both parts of the corner solution model is the traffic variable. Results suggest that an increase in tonnage density both increases the probability (selection equation) and the size of reinvestment (outcome equation). Including the quality variables in the model estimation results in a cost elasticity (Eq. 4) at 0.5665 (std. err. 0.1691) with respect to traffic. This suggests that a 1 per cent increase in tonnage density leads to a 0.57 per cent increase in reinvestment costs. The corresponding estimate from Model 1 (excluding quality variables) results in a cost elasticity at 0.3429 (std. err. 0.1199). It is not straightforward to compare these elasticities with previous results in Andersson et al.

[6] Year dummy variables were tested in the corner solution models, with estimates that were jointly significant in the selection equation but not in the outcome equation. The results in the selection equation changes slightly; for example, the elasticity with respect to gross tons increases from 0.2450 (Model 2) to 0.2646. Models excluding year dummy variables is kept in the presentation for comparison with the survival model.

(2012) and Yarmukhamedov et al. (2016). The reason is that their cost data included switches, while these renewals have been deleted from our data set since switches are not renewed at the same time as other parts of the track superstructure.

Table 2. Corner solution model estimation results – selection equation

	Model 1		*Model 2*	
Selection equation	Coef.	Rob. Std. Err.	Coef.	Rob. Std. Err.
Cons.	-0.5249***	0.1210	-0.2461*	0.1437
ln(route_length)	0.3387***	0.0784	0.0979	0.0804
ln(gross_tons)	0.2591***	0.0567	0.3441***	0.0833
[ln(gross_tons)]^2	0.0759*	0.0425	0.0507	0.0549
ln(rail_weight)	-	-	0.6807	0.9861
[ln(rail_weight)]^2	-	-	-24.6527*	12.8699
ln(quality_class)	-	-	0.0569	0.2461
[ln(quality_class)]^2	-	-	-0.4929	0.6595
ln(joints)	-	-	0.4396***	0.0880
[ln(joints)]^2	-	-	0.1529***	0.0543
ln(gross_tons)ln(rail_weight)	-	-	0.8370	0.6647
ln(gross_tons)ln(quality_class)	-	-	-0.1084	0.1724
ln(rail_weight)ln(quality_class)	-	-	0.3395	1.9854
D.Concrete_sleepers	-	-	-0.2624**	0.1267
ln(gross_tons)D.Concrete_sleepers	-	-	-0.1904*	0.1065
D.Station_section	0.3580*	0.1902	-0.1697	0.1763
D.north_region	-0.0794	0.1678	0.0813	0.1780
D.central_region	0.0547	0.1342	-0.0110	0.1359
D.south_region	0.1792	0.1454	0.3100**	0.1373
D.east_region	0.1374	0.1484	0.1981	0.1446
AIC	3472.546	-	3373.560	-

***, **, * Significant at 1%, 5%, and 10%, respectively.

Table 3. Corner solution model estimation results – outcome equation

	Model 1		*Model 2*	
Outcome equation	Coef.	Rob. Std. Err.	Coef.	Rob. Std. Err.
Cons.	13.6492***	0.2207	14.8207***	0.3391
ln(route_length)	0.6230***	0.1540	0.3423**	0.1610
ln(gross_tons)	0.2919**	0.1307	0.5513***	0.1330
[ln(gross_tons)]^2	0.3171**	0.1325	0.3144***	0.1165
ln(rail_weight)	-	-	-0.4547	1.6932
ln(quality_class)	-	-	0.1851	0.6944
[ln(quality_class)]^2	-	-	-1.1547	1.4509
ln(joints)	-	-	0.4602***	0.1525
[ln(joints)]^2	-	-	0.1258	0.1274
ln(gross_tons)ln(quality_class)	-	-	0.8144**	0.3628
ln(gross_tons)ln(joints)	-	-	-0.2869***	0.1031
ln(quality_class)ln(joints)	-	-	-0.9230***	0.3191
D.Concrete_sleepers	-	-	-1.2971***	0.3023
D.Station_section	0.5217	0.3874	-0.4437	0.3947
D.north_region	0.0167	0.4187	-0.3795	0.5136
D.central_region	-0.7025***	0.2702	-0.9376***	0.2741
D.south_region	-0.3022	0.2884	0.0557	0.2972
D.east_region	-0.6148**	0.3002	-0.3695	0.2910
Sigma	2.6146***	0.0621	2.5339***	0.0628
AIC	4587.697	-	4545.526	-

***, **, * Significant at 1%, 5%, and 10%, respectively.

4.2 Parametric survival model results

The results of the Weibull regression models are presented in Table 4. We first note that the estimates in these models are expected to have the opposite sign compared to the estimates in the probit model (the selection equation, i.e. the first part of the corner solution model). That is, the AFT model estimates show the effect on the survival time of the tracks, while the probit estimates show the effect on the probability of a renewal.

The coefficients for all the quality variables are statistically significant in Model 3. The estimates for traffic are similar between Model 3 and 4, while the coefficients for station sections and regions change considerably. We prefer the Model 4 results, as Model 3 seems to suffer from omitted variable bias.

The first order coefficient for rail weight is negative and the interaction term with gross tons is positive and statistically significant. Calculating the deterioration elasticities with respect to rail weight confirms intuition in that the heaviest rails (60 kg per meter) increases the life length of the track, while lighter rails decreases the life length.

Table 4. Survival model estimation results.

	Model 3		*Model 4*	
	Coef.	Std. Err.	Coef.	Std. Err.
Constant	3.8288***	0.0723	3.6580***	0.0760
ln(segment_length)	0.0071	0.0086	0.0046	0.0095
ln(gross_tons)	-0.2193***	0.0134	-0.2319***	0.0198
[ln(gross_tons)]^2	-0.0935***	0.0076	-0.0806***	0.0073
ln(rail_weight)	-	-	-1.1909***	0.2259
[ln(rail_weight)]^2	-	-	3.7723	2.8098
ln(quality_class)	-	-	-0.2786***	0.0480
[ln(quality_class)]^2	-	-	-0.3656***	0.1094
Joints	-	-	-0.00430**	0.00175
Joints^2	-	-	0.00003	0.00002
ln(gross_tons)ln(quality_class)	-	-	0.0485**	0.0231
ln(gross_tons)ln(rail_weight)	-	-	0.2310*	0.1285
ln(quality_class)ln(rail_weight)	-	-	-1.3760***	0.3757
D.concrete_sleepers	-	-	0.1489***	0.0327
ln(gross tons)D.concrete_sleepers	-	-	0.1066***	0.0255
D.station_section	-0.2291***	0.0513	-0.0222	0.0536
D.west_region	Ref. category		Ref. category	
D.north_region	-0.1720***	0.0386	0.0018	0.0406
D.central_region	0.0481	0.0376	0.2020***	0.0389
D.south_region	-0.0130	0.0362	-0.0336	0.0344
D.east_region	0.0550	0.0401	0.0777**	0.0391
Shape parameter *p*	3.3093		3.5062	

***, **, * Significant at 1%, 5%, and 10%, respectively.

Furthermore, the first and second order coefficients for quality class are negative. This variable also has interaction terms and calculating the deterioration elasticities with respect to quality class indicates that the lowest values (high maximum speed, implying stricter requirements on track geometry) increases the track's life length, while we have the opposite effect for higher values of quality class. Hence, lower line-speed indicates that the track's remaining life time is lower, which suggests that these are tracks with poor quality (for example with respect to track geometry) and need to be renewed. Specifically, it is not a lower line-speed that causes the reduction in expected life, it is rather a poor quality that require low line-speeds, indicating that the remaining life time of the track is shorter.

The number of joints also decrease the track's life time, while we have the opposite effect for tracks with concrete sleepers. Moreover, the results show that rails in the central and east regions have longer life times compared to the west region.

Turning to the estimates of main interest, the results in Model 4 suggest that an 1% increase in traffic density leads to a 0.23 per cent decrease in rail service life time at the sample median. The average elasticity with respect to traffic for the entire sample (i.e. including the second order effect and the interaction terms) corresponds to a 0.10 per cent decrease in the track's life time.

As expected, the shape parameter *p* (3.5062) in Table 4 indicates that the Weibull hazard is monotonically increasing, i.e. the need to renew the tracks accelerates over time as traffic accumulates, as depicted in Figure 1.

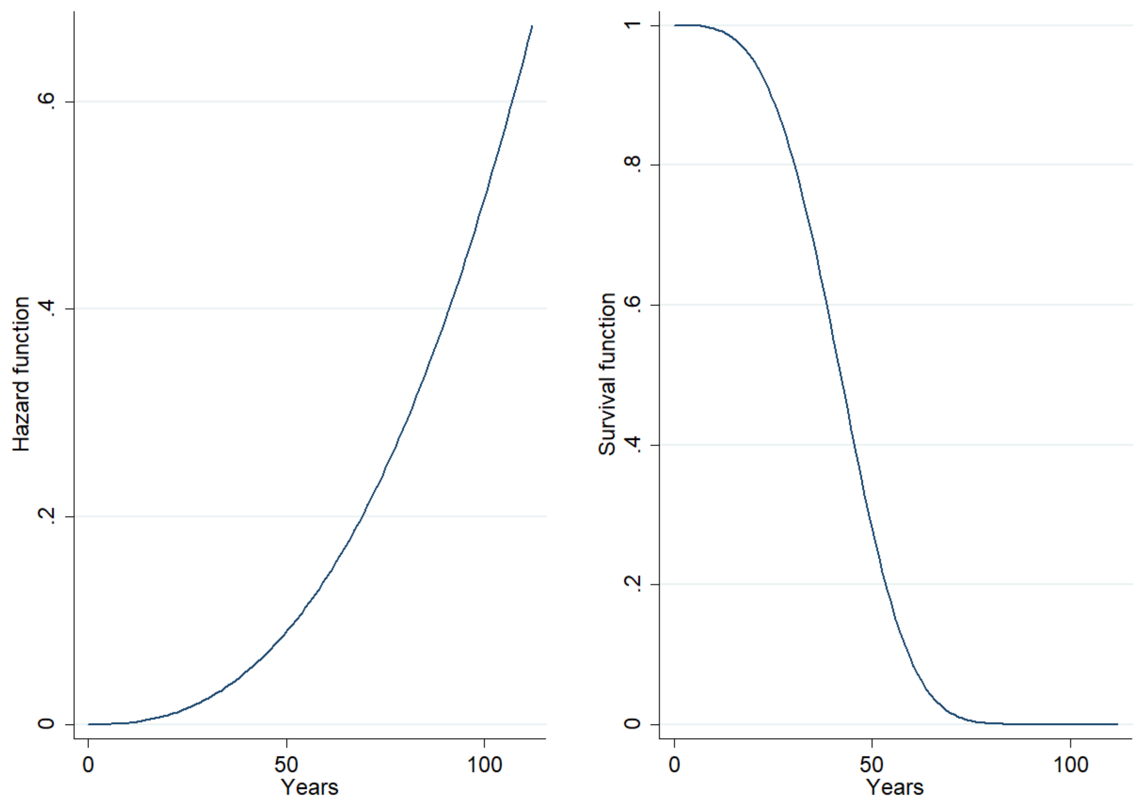

Figure 1. Weibull regression model: hazard and survival functions.

4.3 Marginal costs and result discussion

Equation 3 is used for calculating the marginal cost from the corner solution model results, and each section's share of total ton-km are used as weights (Eq. 5). The weighted marginal cost is SEK 0.0103 per gross ton-km. For the survival model, we use equation 12 to calculate the marginal cost and each segment's share of total ton-km are used as weights (Eq. 5). This results in a weighted marginal cost at SEK 0.0023 per gross ton-km, which is significantly lower than the corner solution results.

However, we get a weighted marginal cost at SEK 0.0041 when we divert slightly from equation 12 (suggested by Andersson et al. (2016), which uses a network average cost) and define the average cost as the sum of all renewal costs *for each track section* over the observation period divided by *each track section's* length of renewed track over the same observation period (a track section specific average cost is also used for the corner solution model results).

A visual inspection of the estimated marginal costs (not weighted) from the models show that the survival model generates a significant number of negative costs (yet, these concern segments with a low traffic volume), while the marginal costs from the corner solution model have marginal costs above zero, which tend to fall sharply with traffic. See Figure 2. The marginal costs from the corner solution have a similar shape as the marginal rail infrastructure maintenance costs for wear and tear (see for example Wheat et al. (2009)).

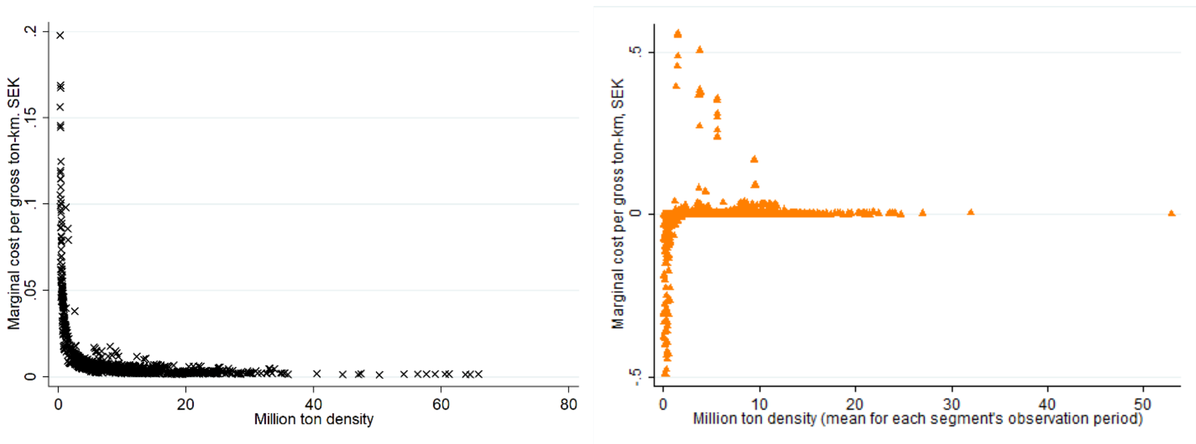

Figure 2. Marginal costs from the corner solution model (left) and survival analysis (right - note that the x-axis is shifted due to the negative marginal cost estimates)

The major difference between the estimated marginal costs seems to be present for low levels of traffic. Still, the weighted marginal cost from the survival model is significantly smaller than the marginal cost from the corner solution model. A large difference is also found when comparing the weighted marginal cost estimates in Andersson et al. (2012) and Andersson et al. (2016), which were SEK 0.009 and 0.002 in the corner solution and survival model, respectively.

What causes this major difference in estimated marginal costs?

From a methodological perspective, survival and corner solution models differ. The former approach estimates how an increase in traffic leads to shorter rail life time, while the latter estimates how traffic results in higher probability and size of reinvestments. A similarity between the models emerge when we reparametrize the AFT estimates to proportional hazard estimates, which then indicates the impact traffic has on the hazard rate, i.e. the risk of reinvestment. That is, the proportional hazard model and the first part of the corner solution model (probit regression) are similar regarding the estimation of the impact traffic has on the probability of observing a reinvestment. Formally, following Doksum and Gasko (1990), the probability that a track with a vector of characteristics x_k will be replaced before time t (=A_t), is written as $\Pr[I(A_t) = 1|X_k = x_k]$. In the survival model, we consider the same track and the probability that it will be replaced before time t, which is the distribution function $F(t|x_k) = \Pr(T \leq t|x_k)$, $t \geq 0$ and T is survival time. With a fixed t, then $F(t|x_k) = \Pr[I(A_t) = 1|X_k = x_k]$.

Therefore, the major difference is that the corner solution model estimates how traffic affects both the probability and size of a reinvestment, while the survival model only considers the probability/risk of a reinvestment. Hence, the first part of the corner solution model should generate results that are closer to the survival model. In fact, the elasticity with respect to gross tons in the probit model is 0.2450, while the corresponding estimate from the survival model is 0.3514, which is the AFT estimate (-0.1002) reparametrized to its proportional hazard form using $-p\beta_{AFT} = \beta_{PH}$. The weighted marginal cost per gross ton-km in the probit model is SEK 0.0019, which is closer to the corresponding costs from the survival model (SEK 0.0023 using a network average cost and SEK 0.0041 using a track section specific average cost).

However, in this comparison, we do need to acknowledge that there are differences in the data samples used in the model estimations; the survival model uses more disaggregate information than the corner solution, and this information is also converted to a cross-section of track individuals, which is not the case for the corner solution model (see section 3).[7] Moreover, the observations of reinvestments are treated differently in the two approaches. The survival approach models duration in which we assume the error terms to be Weibull distributed, while the probit model considers reinvestment counts with normally distributed error terms. Specifically, the survival model considers that the probability of reinvestment will vary with respect to the age of the tracks, while the probit model considers this probability to be invariant to the age of the tracks. Still, this does not seem to be a problem in our probit regression; including a rail age variable does not change the results.

Given that both approaches have similarities with respect to the impact traffic has on the occurrence of reinvestments, we need to ask why we should prefer one before the other? One crucial aspect in addressing this question is that track renewals are not homogenous. Except for variations in length, project costs may differ in a non-stochastic way. Using the two-part approach makes it possible to pick up any differences in costs related to the second step's explanatory variables. In general, when the event (in our case, the reinvestment) can vary continuously in "severity", the corner solution model is preferable to survival analysis. We thus prefer the corner solution model when analyzing the marginal cost of track reinvestments with respect to traffic.

5. Conclusion

The efficient use of railway infrastructure requires a charge that is based on the marginal cost of using that infrastructure. Usage related marginal costs are obtained using econometric techniques, where a traffic variable explains the reinvestment process based on the methodology under consideration. The

[7] However, estimating a probit model on the survival data generates an elasticity with respect to gross tons at 0.4138, while the corresponding estimate from the survival model is 0.3514 (the AFT estimate -0.1002 reparametrized to its proportional hazard form). Thus, the models produce rather similar elasticity estimates for traffic when using the same data set.

present paper has systematically compared two methodologically different approaches that explain the reinvestment process.

Using a parametric survival model, where rail survival time is modelled as a function of traffic and other network characteristics, it is necessary to have access to data which identifies the timing of the original construction year (or the year of the previous renewal) and a number of renewals during the observation period of the data. This makes it possible to measure the rail survival time. Non-renewed track segments over the observation period are denoted as censored observations (tracks that are still "alive"), while renewed segments are said to be uncensored. A deterioration elasticity with respect to traffic is estimated and used in the marginal cost estimation.

The second approach, the corner solution (or two-part) model, uses traffic and other covariates to explain the decision to make a reinvestment and the size of the reinvestment as two separate processes. The first part of the two-stage model uses the entire sample for estimating a probit model, while the second part uses the observations with positive reinvestment costs for estimating a truncated regression model. The traffic estimates from both models are used in the calculation of the marginal cost.

The two approaches are applied on 18 years of information about the Swedish rail network to illuminate the difference in results when marginal renewal costs are estimated. The effect of the traffic variable in both approaches is of key interest as it provides elasticity estimates required to calculate the marginal cost per ton-km.

This elasticity differs substantially between the model approaches. The major reason is that the survival model only considers the risk of reinvestment which thereafter is transferred to a marginal cost outside the model. The corner solution approach also includes the impact traffic has on the size of the reinvestment cost.

In this study, the marginal cost per gross ton-km is more than two times higher when using the corner solution rather than the survival model. Similar differences can be found when comparing results from previous studies using either approach. Considering that there are systematic variations in the size of the reinvestment cost, which partly can be explained by traffic, a proper methodology needs to capture these variations. This is the case for the corner solution model, but not the survival model.

All in all, the corner solution model is less restrictive in capturing the behavior of the IM, compared to the parametric survival model. This implies that the results from the corner solution model are suggested to further reference.

Finally, we can conclude that the two approaches generate different requirements on the level of detail of data, which is important from a practical point of view in the estimation of marginal costs. The survival approach requires disaggregate data on the investment and reinvestment dates of the assets, while the cost data do not have to be at the same level of detail. In fact, costs can be a network average, even though it is not ideal. The corner solution model, on the other hand, requires more disaggregate cost data, while information on the life times of the different assets is not required. If aggregate cost data make the survival approach the only option, the IM should acknowledge that this approach may underestimate the impact traffic has on costs.

The access to detailed information about the railway network has highlighted an aspect which is important for future empirical applications of the corner solution model. This refers to the necessity to have information about track quality variables such as quality class (line-speed), rail weight and the use of concrete sleepers. A comparison has established that the inclusion of quality variables has a large impact on the estimates for traffic. Considering that, for example, the correlation coefficient between the logarithm of quality class and the logarithm of gross tons is -0.5357, omitting the former variable implies that the estimate for the latter will be biased (and vice versa).

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